Topological Matter and Phases: the Nobel Prize in Physics 2016

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Summary

- Nobel in Physics 2016: Thouless, Haldane and Kosterlitz
- Bosons and fermions
- Superconductivity and superfluidity
- Topology in Physics: quantized vortices
- Bose-Einstein condensation and Mermin-Wagner theorem
- 2D systems: topological phase transition of Kosterlitz-Thouless
- Research activity in our Department on topological matter
- Conclusions

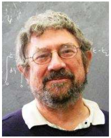
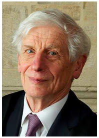
Nobel in Physics 2016: Thouless, Kosterlitz and Haldane

The Nobel Prize in Physics 2016 was divided, one half awarded to **David J. Thouless**, the other half jointly to **F. Duncan M. Haldane** and **J. Michael Kosterlitz**

"for theoretical discoveries of topological phase transitions and topological phases of matter".

Nobel in Physics 2016: Thouless, Kosterlitz and Haldane

The Nobel winners have used advanced mathematical methods to explain strange phenomena in unusual phases (or states) of matter, such as superconductors, superfluids or thin magnetic films.

Kosterlitz (Brown Univ) and **Thouless** (Washington Univ) have studied phenomena that arise in a flat world on surfaces or inside extremely thin layers that can be considered two-dimensional.

Haldane (Princeton Univ) has also studied matter that forms threads so thin they can be considered one-dimensional.

Bosons and fermions

Any particle has an intrinsic angular momentum, called spin $\vec{S} = (S_x, S_y, S_z)$, characterized by two quantum numbers s ed m_s, where for s fixed one has $m_s = -s, -s+1, ..., s-1, s$, and in addition

$$S_z = m_s \hbar \ ,$$

with $\hbar$ ($1.054 \cdot 10^{-34}$ Joule×seconds) the reduced Planck constant.
All the particles can be devided into two groups:
– **bosons**, characterized by an integer s:

$$s = 0, 1, 2, 3, ...$$

– **fermions**, characterized by a half-integer s:

$$s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, ...$$

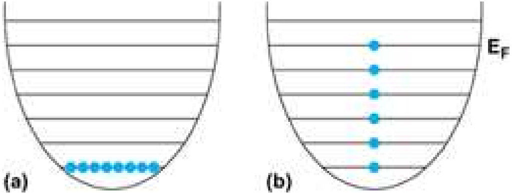
Examples: the photon is a boson ($s = 1$, $m_s = -1, 1$), while the electron is a fermion ($s = \frac{1}{2}$, $m_s = -\frac{1}{2}, \frac{1}{2}$).
Among "not elementary particles": helium ${}^4_2$He is a boson ($s = 0$, $m_s = 0$), while helium ${}^3_2$He is a fermion ($s = \frac{1}{2}$, $m_s = -\frac{1}{2}, \frac{1}{2}$).

Bosons and fermions

A fundamental experimental result: identical bosons and identical fermions have a very different behavior!!

– Identical bosons can occupy the same single-particle quantum state, i.e. thay can stay together; if all bosons are in the same single-particle quantum state one has Bose-Einstein condensation.

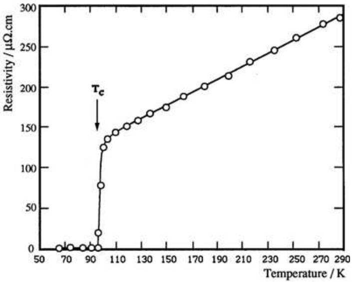
– Identical fermions CANNOT occupy the same single-particle quantum state, i.e. they somehow repel each other: Pauli exclusion principle.



Identical bosons (a) and identical spin-polarized fermions (b) in a harmonic trap.

Superconductivity and superfluidity

Superconductivity is a phenomenon of exactly zero electrical resistance and expulsion of magnetic flux fields occurring in certain materials when cooled below a characteristic critical temperature T_c.
It was discovered in 1911 by **Heike Kamerlingh Onnes** (Nobel 1913).



In 1957 **John Bardeen**, **Leon Cooper** and **Robert Schrieffer** suggested that in superconductivity, due to the ionic lattice, pairs of electrons behave like bosons (Nobel 1972), as somehow anticipated in 1950 by **Lev Landau** (Nobel 1962) and **Vitaly Ginzburg** (Nobel 2003).

Superconductivity and superfluidity

Critical temperature T_c of some superconductors at atmospheric pressure.

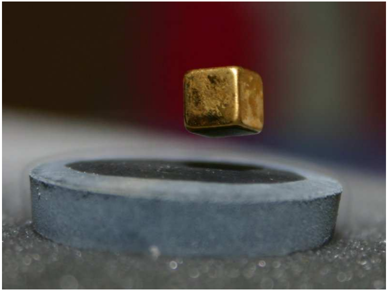
Material	**Symbol**	T_c **(Kelvin)**
Aluminium	${}^{27}_{13}Al$	1.19
Tin	${}^{120}_{50}Sn$	3.72
Mercury	${}^{202}_{80}Hg$	4.16
Lead	${}^{208}_{82}Pb$	7.20
Neodymium	${}^{142}_{60}Nb$	9.30

In 1986 **Karl Alex Müller** and **Johannes Georg Bednorz** discovered high-T_c superconductors (Nobel 1987). These ceramic materials (cuprates) can reach the critical temperature of 133 Kelvin.

For these high-T_c superconductors the mechanisms which give rise to pairing of elettrons are not fully understood.

Superconductivity and superfluidity

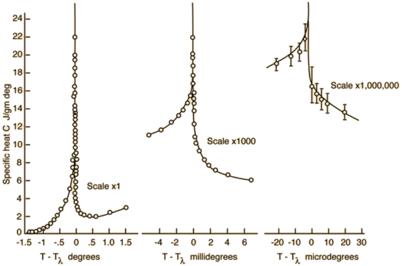
Superconductors have interesting properties. For instance the levitation of a magnetic material over a supercondutor (Meissner effect).



Some technological applications of superconductors:
– MAGLEV trains, based on magnetic levitation (mag-lev);
– SQUIDS, devices which measure extremely week mgnetic fields;
– very high magnetic fields for Magnetic Resonance in hospitals.

Superconductivity and superfluidity

Superfluidity is the characteristic property of a fluid with zero viscosity, which therefore flows without loss of kinetic energy.
It was discovered in 1937 by **Pyotr Kapitza** (Nobel 1978), who found that, at atmospheric pressure, below $T_\lambda = 2.16$ Kelvin helium 4 (^{4}He) not only remains liquid but it also shows zero viscosity. Moreover at T_λ the specific heat diverges.



In 1938 **Fritz London** gave a first theoretical explanation of superfluidity of helium 4 on the basis of Bose-Einstein condensation (BEC), but **Laszlo Tisza** and **Lev Landau** (Nobel 1962) ware able to describe superfluidity without invoking BEC.

Superconductivity and superfluidity

A **fluid** can be described by the Navier-Stokes equations of hydrodynamics

$$\begin{aligned} \frac{\partial}{\partial t} n + \nabla \cdot (n\mathbf{v}) &= 0 \, , \\ m \frac{\partial}{\partial t} \mathbf{v} - \eta \, \nabla^2 \mathbf{v} + \nabla \left[\frac{1}{2} m v^2 + U_{ext} + \mu(n) \right] &= m \, \mathbf{v} \wedge (\nabla \wedge \mathbf{v}) \, , \end{aligned}$$

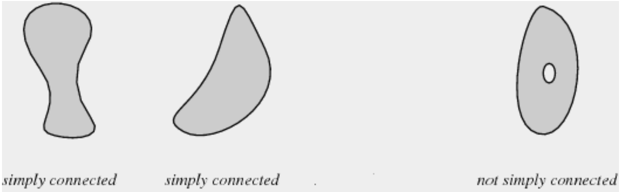
where $n(\mathbf{r}, t)$ is the density field and $\mathbf{v}(\mathbf{r}, t)$ is the velocity field. Here η is the viscosity, $U_{ext}(\mathbf{r})$ is the external potential acting on the particles of the fluid, and $\mu(n)$ is the equation of state of the fluid.
A superfluid is characterized by zero viscosity, i.e. $\eta = 0$, and irrotationality, i.e. $\nabla \wedge \mathbf{v} = \mathbf{0}$.
The equations of superfluid hydrodynamics (EoSH) are then

$$\begin{aligned} \frac{\partial}{\partial t} n_s + \nabla \cdot (n_s \mathbf{v}_s) &= 0 \, , \\ m \frac{\partial}{\partial t} \mathbf{v}_s + \nabla \left[\frac{1}{2} m v_s^2 + U_{ext} + \mu(n_s) \right] &= \mathbf{0} \, . \end{aligned}$$

EoSH describe extremely well the superfluid ^{4}He, ultracold gases of alkali-metal atoms, and also several properties of superconductors.

Topology in Physics: quantized vortices

The advanced mathematical methods used by Thouless, Haldane and Kosterlitz are based also on ideas taken from topology, which studies objects that are preserved under continuous deformations.



A **connected domain** is said to be simply connected if any closed curve $\mathcal{C}$ can be shrunk to a point continuously in the set.
If the domain is not simply connected, it is said to be multiply connected.

Roughly speaking, a way to make a connected domanin multiply connected is to introduce **holes**.

Topology in Physics: quantized vortices

In the 1950s **Lars Onsager**, **Richard Feynman** (Nobel 1965), and **Alexei Abrikosov** (Nobel 2003) suggested that for superfluids the circulation of the superfluid velocity field $\mathbf{v}_s(\mathbf{r}, t)$ around a generic closed path $\mathcal{C}$ must be quantized, namely

$$\oint_{\mathcal{C}} \mathbf{v}_s \cdot d\mathbf{r} = \frac{\hbar}{m} \, 2\pi \; q \; ,$$

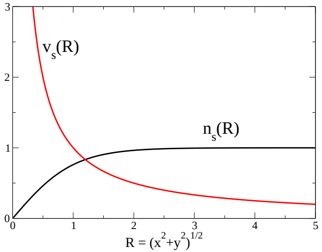
where $\hbar$ is the reduced Planck constant and $q = 0, \pm 1, \pm 2, ...$ is an integer number.
If $q \neq 0$ it means that inside the closed path $\mathcal{C}$ there are topological defects, and the domain where $\mathbf{v}_s$ is well defined is multiply connected.

For a multiply connected domain $\mathcal{D}$, with $\mathbf{r} \in \mathcal{D}$ one gets

$$\boldsymbol{\nabla} \wedge \mathbf{v}_s(\mathbf{r}) = \mathbf{0} \implies \mathbf{v}_s(\mathbf{r}) = \boldsymbol{\nabla}\chi(\mathbf{r}) \text{ with } \chi(\mathbf{r}) \text{ multi-valued scalar field.}$$

Topology in Physics: quantized vortices

A simple example of topological defect is a quantized vortex line along the z axis.



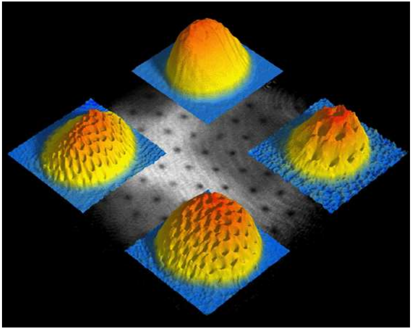
Vortex line: superfluid number density n_s and modulus of the superfluid velocity v_s as a function of the cylindric radial coordinate R.

$$n_s(R) \simeq n_s(\infty) \left(1 - \frac{1}{1 + \frac{R^2}{\xi^2}} \right) \quad \text{and} \quad v_s(R) = \frac{\hbar}{m} \frac{q}{R^2}$$

Clearly at $R = 0$, i.e. at $(x, y) = (0, 0)$, the superfluid velocity is not defined. q is called charge of the vortex and ξ is the healing length.

Topology in Physics: quantized vortices

Nowadays quantized vortices are **observed experimentally** in type-II superconductors, in superfluid liquid helium, and in ultracold atomic gases.



Formation of quantized vortices in a Bose-Einstein condensate of ^{87}Rb atoms. The number of quantized vortices grows by increasing the frequency of rotation of the system [J. R. Abo-Shaeer, C. Raman, J.M. Vogels, W. Ketterle, Science **292**, 476 (2001)]. **Wolfang Ketterle**, with **Eric Cornell** and **Carl Wieman** (Nobel 2001).

Topology in Physics: quantized vortices

The quantization of circulation can be explained assuming that the dynamics of **superfluids** is driven by a complex scalar field

$$\psi(\mathbf{r}, t) = |\psi(\mathbf{r}, t)| \; e^{i\theta(\mathbf{r},t)} \; ,$$

which satisfies the nonlinear Schrödinger equation (NLSE)

$$i\hbar \frac{\partial}{\partial t}\psi = \left[-\frac{\hbar^2}{2m}\nabla^2 + U_{ext} \right] \psi + \mu(|\psi|^2) \; \psi$$

and it is such that

$$n_s(\mathbf{r}, t) = |\psi(\mathbf{r}, t)|^2 \; , \qquad \mathbf{v}_s(\mathbf{r}, t) = \frac{\hbar}{m} \boldsymbol{\nabla}\theta(\mathbf{r}, t) \; .$$

In fact, under these assumptions, NLSE is practically equivalent to EoSH and the multi-valued angle variable $\theta(\mathbf{r}, t)$ is such that

$$\oint_{\mathcal{C}} d\theta = \oint_{\mathcal{C}} \boldsymbol{\nabla}\theta(\mathbf{r}, t) \cdot d\mathbf{r} = 2\pi q \; ,$$

with $q = 0, \pm 1, \pm 2, \ldots$. The quantized vortex line is then obtained with

$$\theta(x, y, z) = q \arctan\left(\frac{y}{x}\right) \; .$$

Topology in Physics: quantized vortices

The NLSE of the complex scalar field $\psi(\mathbf{r}, t)$ of superfluids

$$i\hbar \frac{\partial}{\partial t}\psi = \left[-\frac{\hbar^2}{2m}\nabla^2 + U_{ext} \right] \psi + \mu(|\psi|^2)\ \psi$$

admits the constant of motion (energy of the system)

$$H = \int \ \left\{ \frac{\hbar^2}{2m}|\nabla\psi|^2 + U_{ext}\ |\psi|^2 + \mathcal{E}(|\psi|^2) \right\}\ d^D\mathbf{r}\ ,$$

where $\mu(|\psi|^2) = \frac{\partial \mathcal{E}(|\psi|^2)}{\partial |\psi|^2}$. Taking into account that

$$\psi(\mathbf{r}, t) = n_s(\mathbf{r}, t)^{1/2}\ e^{i\theta(\mathbf{r}, t)} \quad \text{with} \quad \mathbf{v}_s(\mathbf{r}, t) = \frac{\hbar}{m}\boldsymbol{\nabla}\theta(\mathbf{r}, t)$$

one finds

$$\frac{\hbar^2}{2m}|\nabla\psi|^2 = \frac{\hbar^2}{2m}\ n_s\ (\nabla\theta)^2 + \frac{\hbar^2}{8m}\frac{(\nabla n_s)^2}{n_s}\ ,$$

which are phase-stiffness energy and quantum pressure.

Bose-Einstein condensation and Mermin-Wagner theorem

In three spatial dimensions ($D = 3$), the complex scalar field of superfluids is called order parameter of the system and it is often identified as the **macroscopic wavefunction** of Bose-Einstein condensation (BEC), where a macroscopic fraction of particles occupies the same single-particle quantum state.

BEC phase transition: For an ideal gas of non-interacting identical bosons there is BEC only below a critical temperature T_{BEC}. In particular one finds

$$k_B\, T_{BEC} = \begin{cases} \frac{1}{2\pi\zeta(3/2)^{2/3}} \frac{\hbar^2}{m} n^{2/3} & \text{for } D = 3 \\ 0 & \text{for } D = 2 \\ \text{no solution} & \text{for } D = 1 \end{cases}$$

where D is the spatial dimension of the system, n is the number density, and $\zeta(x)$ is the Riemann zeta function.

This result due to Einstein (1925), which says that there is no BEC at finite temperature for $D \leq 2$ in the case of non-interacting bosons, was extended to interacting systems by **David Mermin** and **Herbert Wagner** in 1966.

Bose-Einstein condensation and Mermin-Wagner theorem

For the BEC phase transition, the Mermin-Wagner theorem states that there is no Bose-Einstein condensation at finite temperature in homogeneous systems with sufficiently short-range interactions in dimensions $D \leq 2$.
Given the bosonic quantum field operator $\hat{\phi}(\mathbf{r}, t)$, the theorem implies that at finite temperature T and with $D \leq 2$ one gets

$$\langle \hat{\phi}(\mathbf{r}, t) \rangle_T = 0 \ ,$$

where $\langle ... \rangle_T$ is the thermal average. Instead, with $D = 3$ and $T < T_{BEC}$ one finds

$$\langle \hat{\phi}(\mathbf{r}, t) \rangle_T \neq 0 \ .$$

Only in this case one can make the identification

$$\langle \hat{\phi}(\mathbf{r}, t) \rangle_T = \psi(\mathbf{r}, t) \ ,$$

where $\psi(\mathbf{r}, t)$ is the complex scalar field of superfluidity.

2D systems: Kosterlitz-Thouless transition

Despite the absence of BEC, in 1972 **Kosterlitz** and **Thouless** (but also **Vadim Berezinskii** (1935-1980)) suggested that a 2D fluid can be superfluid below a critical temperature, the so-called Berezinskii-Kosterlitz-Thouless critical temperature T_{BKT}.
They analyzed the 2D XY model, which was originally used to describe the magnetization in a planar lattice of classical spins. The Hamiltonian of the continuous 2D XY model is given by

$$H = \int \frac{J}{2} (\nabla\theta)^2 \; d^2\mathbf{r} \,,$$

where $\theta(\mathbf{r})$ is the angular field and J is the phase stiffness (rigidity).
This Hamiltonian is nothing else than the constant of motion (energy)

$$H = \int \left\{ \frac{\hbar^2}{2m} |\nabla\psi|^2 + \mathcal{E}(|\psi|^2) \right\} \; d^2\mathbf{r}$$

of the 2D NLSE of the complex scalar field $\psi(\mathbf{r}, t)$ of 2D superfluids.
Here $U_{ext} = 0$ and

$$\psi(\mathbf{r}) = n_s^{1/2} \; e^{i\theta(\mathbf{r})}$$

with $J = n_s(\hbar^2/m)$, and neglecting the bulk energy $\mathcal{E}(n_s)L^2$.

Formally, one can rewrite the angular field as follows

$$\theta(\mathbf{r}) = \theta_0(\mathbf{r}) + \theta_v(\mathbf{r}) \ ,$$

where $\theta_0(\mathbf{r})$ has zero circulation (no vortices) while $\theta_v(\mathbf{r})$ encodes the contribution of quantized vortices.
After some manipulations, the 2D XY Hamiltonian can be rewritten as

$$H = \int \frac{J}{2} (\boldsymbol{\nabla}\theta_0)^2 d^2\mathbf{r} + \sum_{i \neq j} V(\mathbf{r}_i - \mathbf{r}_j) q_i q_j - \sum_j \mu_c \, q_j^2$$

where the second term describes quantized vortices located at position $\mathbf{r}_i$ with quantum numbers q_i, interacting through a 2D Coulomb-like potential

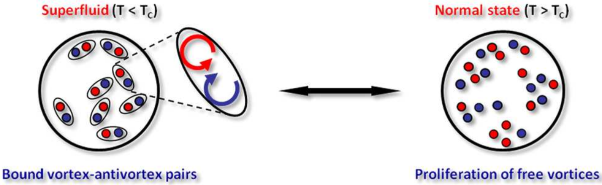
$$V(r) = -2\pi J \ln\left(\frac{r}{\xi}\right) \ ,$$

where ξ is healing length, i.e. the cutoff length defining the vortex core size, and μ_c the energy associated to the creation of a vortex.

2D systems: Kosterlitz-Thouless transition

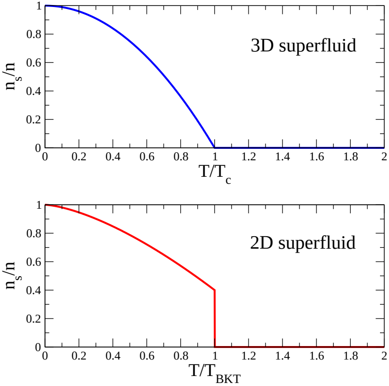
The analysis of **Kosterlitz** and **Thouless** on this 2D Coulomb-like gas of quantized vortices has shown that:

- As the temperature T increases vortices start to appear in vortex-antivortex pairs (mainly with $q = \pm 1$).
- The pairs are bound at low temperature until at the critical temperature $T_c = T_{BKT}$ an unbinding transition occurs above which a proliferation of free vortices and antivortices is predicted.
- The phase stiffness J and the vortex energy μ_c are renormalized.
- The renormalized superfluid density $n_s = J(m/\hbar^2)$ decreases by increasing the temperature T and jumps to zero above $T_c = T_{BKT}$.



2D systems: Kosterlitz-Thouless transition

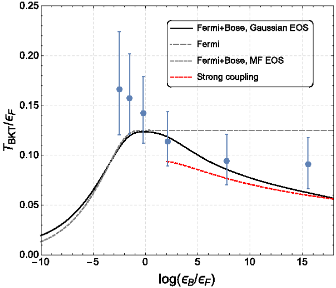
An important prediction of the Kosterlitz-Thouless transition is that, contrary to the 3D case, in 2D the superfluid fraction n_s/n jumps to zero above a critical temperature.



For 3D superfluids the transition to the normal state is a BEC phase transition, while in 2D superfluids the transition to the normal state is something different: a topological phase transition.

Research activity in our Department on topological matter

The Kosterlitz-Thouless (KT) transition has been observed experimentally in various physical systems: thin films of superfluid ^{4}He (1978), thin films of superconductors (1981), quasi-2D bosonic gas of ^{87}Rb atoms (2006).



Very recently the KT transition has been observed also in quasi-2D superfluid fermionic gases made of ^{6}Li atoms in the BCS-BEC crossover [P.A. Murthy et al., PRL **115**, 010401 (2015)]. In the figure our theoretical results of T_{BKT} vs interaction strength compared with experimental data (circles with error bars) [G. Bighin and LS, Phys. Rev. B **93**, 014519 (2016)].

Research activity in our Department on topological matter

In our Department there are various theoretical groups which are investigating topological phase transitions and topological states:

- KT transition in superfluids and high-Tc superconductors (Dell'Anna, Marchetti, Salasnich)
- Topological defects in liquid helium and atomic gases (Ancilotto, Salasnich, Silvestrelli)
- Topological phases in 1D systems (Dell'Anna)

Further research involving topology in physics:

- Topology in soft matter: knots and links in biomolecules, liquid crystals (Baldovin, Baiesi, Orlandini, Seno, Stella, Trovato)
- Topological properties of networks (Maritan, Suweis)
- Topology in string theory (Martucci, Matone)

Conclusions

At the end of this talk I hope you may comment using the words of Enrico Fermi:

"Before I came here I was confused about this subject.
Having listened to your lecture I am still confused.
But on a higher level.

THANK YOU VERY MUCH FOR YOUR ATTENTION!

Slides online: http://materia.dfa.unipd.it/salasnich/talk-toponobel.pdf