Static and dynamics of attractive BECs in an axial optical lattice

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Partial support: CARIPARO Foundation and GNFM-INdAM

Talk at Toledo, April 02, 2008

Summary

- Confined attractive BEC in an axial optical lattice
- Gaussian variational approach
- The nonpolynomial Schrödinger equation (NPSE)
- Ground-state bright solitons and their collapse
- Comparison between NPSE and 3D GPE
- Dynamics of kicked bright solitons (preliminar results)
- Conclusion

Confined attractive BEC in an axial optical lattice

The energy-per-atom (E) of the self-attractive BEC described by the mean-field stationary wave function, $\psi(\mathbf{r})$, in the presence of the strong transverse harmonic confinement with frequency $\omega_\perp$, acting in the plane of (x, y), is

$$E = \int d\mathbf{r}\,\psi^*(\mathbf{r})\left[-\frac{1}{2}\nabla^2 + \frac{1}{2}(x^2+y^2) + V(z) - \pi g|\psi(\mathbf{r})|^2\right]\psi(\mathbf{r})\,. \qquad (1)$$

Here, the OL potential acting along axis z is

$$V(z) = -V_0 \cos(2k_L z)\,, \qquad (2)$$

with

$$k_L = \frac{2\pi}{\lambda}\sin\left(\frac{\theta}{2}\right)\,, \qquad (3)$$

where λ is the wavelength of two laser beams with angle θ between them that create the AOL.

We assume normalization

$$\int |\psi(\mathbf{r})|^2 d\mathbf{r} \equiv 1, \tag{4}$$

then

$$g \equiv \frac{2|a_s|N}{a_\perp} \tag{5}$$

is the adimensional strength of the self-attraction, with negative scattering length of atomic collisions a_s, and the number of atoms in the condensate, N.

Lengths are measured in units of the transverse harmonic length,

$$a_\perp = \sqrt{\frac{\hbar}{m\omega_\perp}}\ , \tag{6}$$

with m is the atomic mass, and the depth of the potential, V_0, is taken in units of $\hbar\omega_\perp$.

Mean-field regime: In our investigation the maximum lattice height is $V_0/E_R = 4$, with $E_R = k_L^2/2$ the recoil energy, while the maximum value of the gas parameter is $n^{1/3}|a_s| = 0.25$, with n the 3D local density.

Gaussian variational approach

To predict solitons in an approximate analytical form, we use the 3D Gaussian ansatz,

$$\psi(\mathbf{r}) = \frac{1}{\pi^{3/4}\sigma\eta^{1/2}} \exp\left\{-\frac{(x^2+y^2)}{2\sigma^2} - \frac{z^2}{2\eta^2}\right\}, \tag{7}$$

where σ and η are, respectively, the transverse width and axial length of the localized pattern. Inserting this ansatz into Eq. (1), we obtain

$$E = \frac{1}{2}\left(\frac{1}{2\eta^2} + \frac{1}{\sigma^2} + \sigma^2\right) - \frac{g}{2\sqrt{2\pi}}\frac{1}{\sigma^2\eta} - V_0 \exp\left(-k_L^2\eta^2\right), \tag{8}$$

We look for values of σ and η that minimize energy E and get

$$-\frac{1}{\eta^3} + \frac{g}{(2\pi)^{1/2}}\frac{1}{\sigma^2\eta^2} + 4V_0 k_L^2 \eta \exp\left(-k_L^2\eta^2\right) = 0, \tag{9}$$

$$-\frac{1}{\sigma^3} + \sigma + \frac{g}{(2\pi)^{1/2}}\frac{1}{\sigma^3\eta} = 0, \tag{10}$$

which can be solved numerically. These solutions yield a ground state, i.e., a minimum of energy, only if the curvature of the energy dependence, $E(\eta,\sigma)$, is positive.

Nonpolynomial Schrödinger equation (NPSE)

A more accurate analysis of the present setting may be performed using the variational ansatz

$$\psi(\mathbf{r}) = \frac{1}{\pi^{1/2}\sigma(z)} \exp\left\{-\frac{(x^2+y^2)}{2\sigma(z)^2}\right\} f(z) \; . \qquad (11)$$

Inserting this expression into the 3D energy functional one finds

$$E = \int dz\, f^*(z) \left[-\frac{1}{2}\frac{d^2}{dz^2} + \frac{1}{2}\left(\frac{1}{\sigma(z)^2} + \sigma(z)^2\right) + V(z) - \frac{1}{2}\frac{g}{\sigma(z)^2}|f(z)|^2\right] f(z) \; . \qquad (12)$$

Minimizing this energy, one arrives at the following equations for real functions $f(z)$ and $\sigma(z)$

$$\left[-\frac{1}{2}\frac{\partial^2}{\partial z^2} - V_0 \cos(2k_L z) + \frac{1-(3/2)g|f(z)|^2}{\sqrt{1-g|f(z)|^2}}\right] f(z) = \mu f(z) \; , \qquad (13)$$

$$\sigma(z) = \left(1 - g|f(z)|^2\right)^{1/4} . \qquad (14)$$

Eq. (13) is the stationary NPSE* with axial periodic potential.

*L.S., Laser Phys. **12**, 198 (2002); L.S., A. Parola, and L. Reatto, PRA **65**, 043614 (2002).

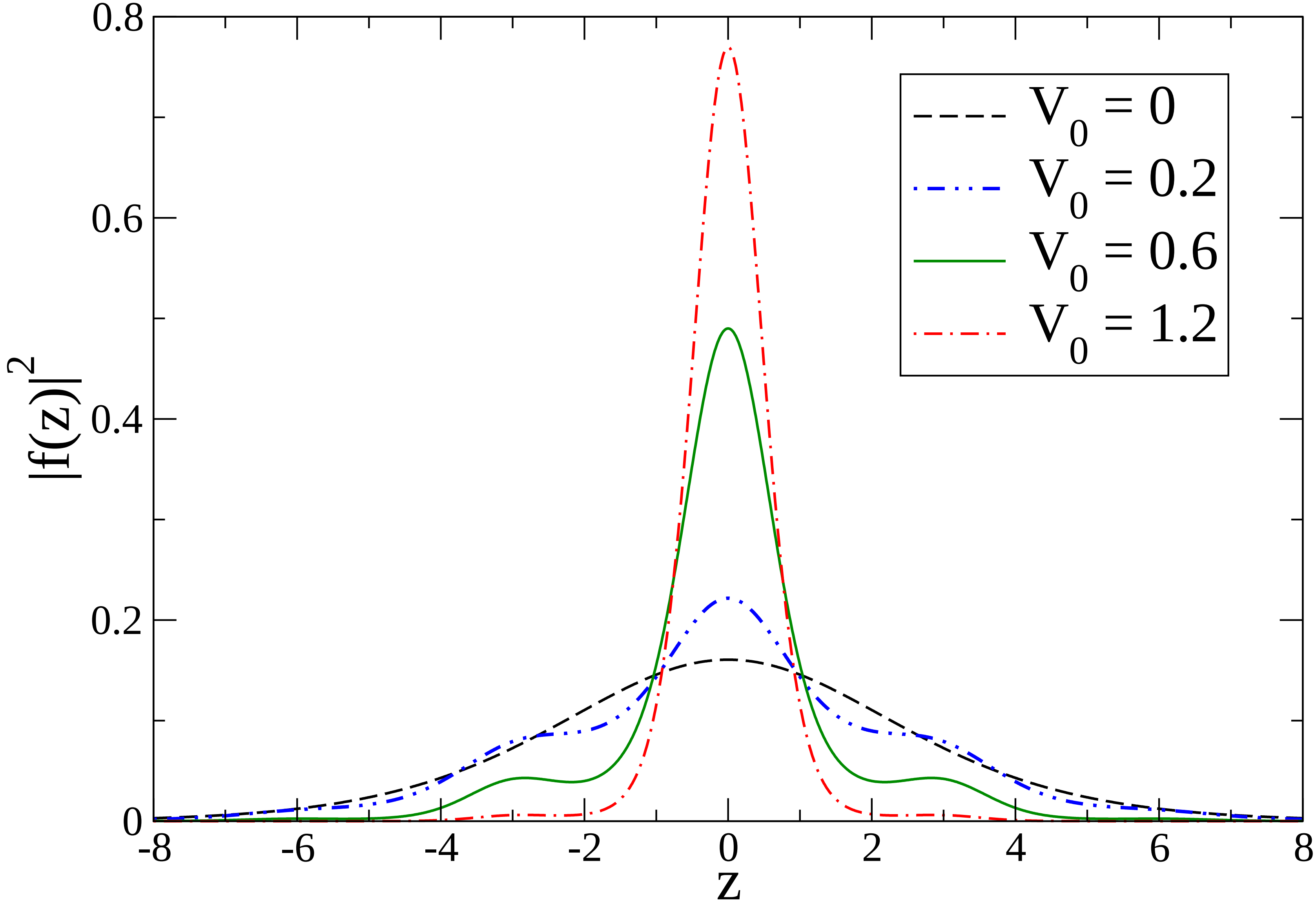

FIG. 1: The axial density profile, $|f(z)|^2$, of the soliton in periodic potential, with $k_L = 1$ and four different values of V_0. The self-attraction strength is fixed at $g = 0.5$. From: L.S., A. Cetoli, B.A. Malomed, and F. Toigo, PRA **75**, 033622 (2007).

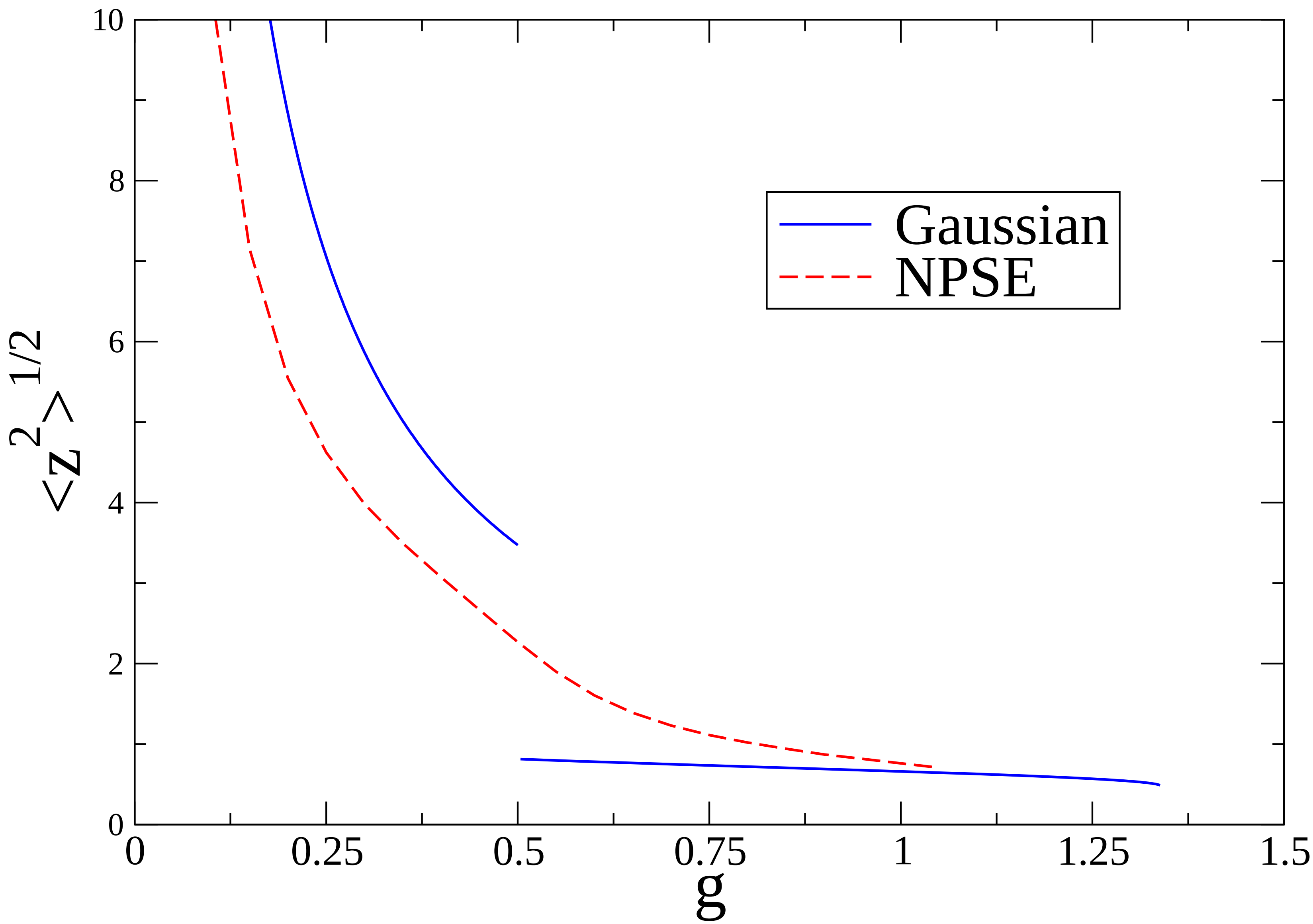

FIG. 2: Axial length of the ground-state bright soliton, $\langle z^2 \rangle^{1/2}$, as a function of self-attraction strength g, for $V_0 = 0.4$ and $k_L = 1$. Displayed are results provided by the Gaussian variational and by the NPSE.

From: L.S., A. Cetoli, B.A. Malomed, and F. Toigo, PRA **75**, 033622 (2007).

V_0	g_c	$\sqrt{\langle z^2 \rangle}$	$\sigma(0)$
0	1.33	0.91	0.75
0.1	1.26	0.77	0.68
0.5	1.07	0.64	0.61
1	0.96	0.50	0.60
2	0.85	0.41	0.57

TABLE 1: The critical value of the self-attraction strength, g_c, and the corresponding values of the axial length, $\sqrt{\langle z^2 \rangle}$, and minimal transverse width, $\sigma(0)$, of the soliton in the periodic potential, $V(z) = -V_0 \cos(2k_L z)$, with $k_L = 1$, for different values of V_0, as found from numerical solution of the NPSE.

From: L.S., A. Cetoli, B.A. Malomed, and F. Toigo, PRA **75**, 033622 (2007).

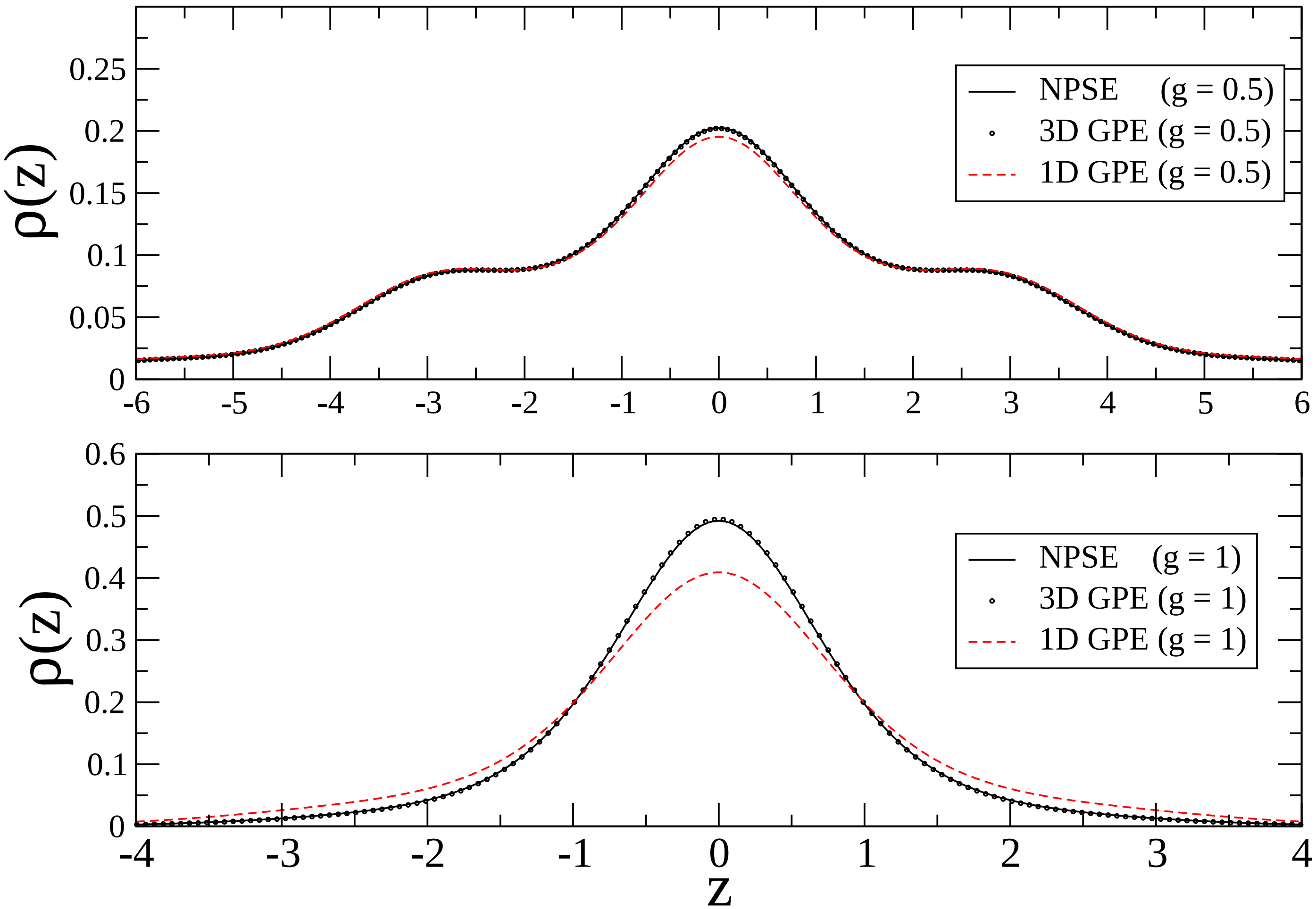

FIG. 3: The axial density profile, $\rho(z)$, of the soliton in potential, with $k_L = 1$ and $V_0 = 0.2$. Comparison between results provided by the different equations: NPSE, 3D GPE, and 1D GPE. In the case of the 3D equation, the axial density is defined as $\rho(z) = \int\int |\psi(\mathbf{r})|^2 dxdy$, while in the other cases it is simply $|f(z)|^2$.

From: L.S., A. Cetoli, B.A. Malomed, and F. Toigo, PRA **75**, 033622 (2007).

Dynamics of kicked bright solitons

To initiate the dynamics, we multiply the stationary solution $f_0(z)$ of NPSE with AOL by $\exp(ipz)$, i.e., use initial conditions

$$f(z) = f_0(z) \ \exp(ipz) \ , \tag{15}$$

where p is the momentum of the imposed kick.

We solved the full time-dependent NPSE,

$$i\,\frac{\partial f(z,t)}{\partial t} = \left[-\frac{1}{2}\frac{\partial^2}{\partial z^2} - V_0 \cos\left(2k_L z\right) + \frac{1-\frac{3}{2}g|f(z,t)|^2}{\sqrt{1-g|f(z,t)|^2}} \right] f(z,t), \tag{16}$$

with the initial condition of Eq. (15), by using a Crank-Nicholson predictor-corrector algorithm in real time.

Note that configuration (15) can be created experimentally by means of the so-called phase-imprinting technique[†].

[†]J. Denschlag *et al.*, Science **287**, 97 (2000).

To characterize the motion of the kicked soliton we calculate the average axial position of the soliton,

$$z_0(t) = \int |f(z,t)|^2 \ dz \ , \tag{17}$$

and its average squared width,

$$\langle z^2(t) \rangle = \int dz \ (z - z_0(t))^2 \ |f(z,t)|^2 \ dz \ . \tag{18}$$

These expressions are not divided by the norm of wave function, as it is fixed to be 1.

We introduce also the following *effective Shannon entropy*,

$$S(t) = -\sum_n A_n(t) \frac{\ln A_n(t)}{\ln(N_{\text{cell}})} \ , \tag{19}$$

where

$$A_n(t) = \int_{n^{th} cell} |f(z,t)|^2 \ dz \tag{20}$$

is the share of the norm located, at time t, within the n^{th} lattice cell, and N_{cell} is the total number of cells ($N_{cell} = 32$).
Maximum entropy, $S = 1$: the matter is distributed uniformly;
minimum entropy, $S = 0$: the entire norm is concentrated in a single cell.

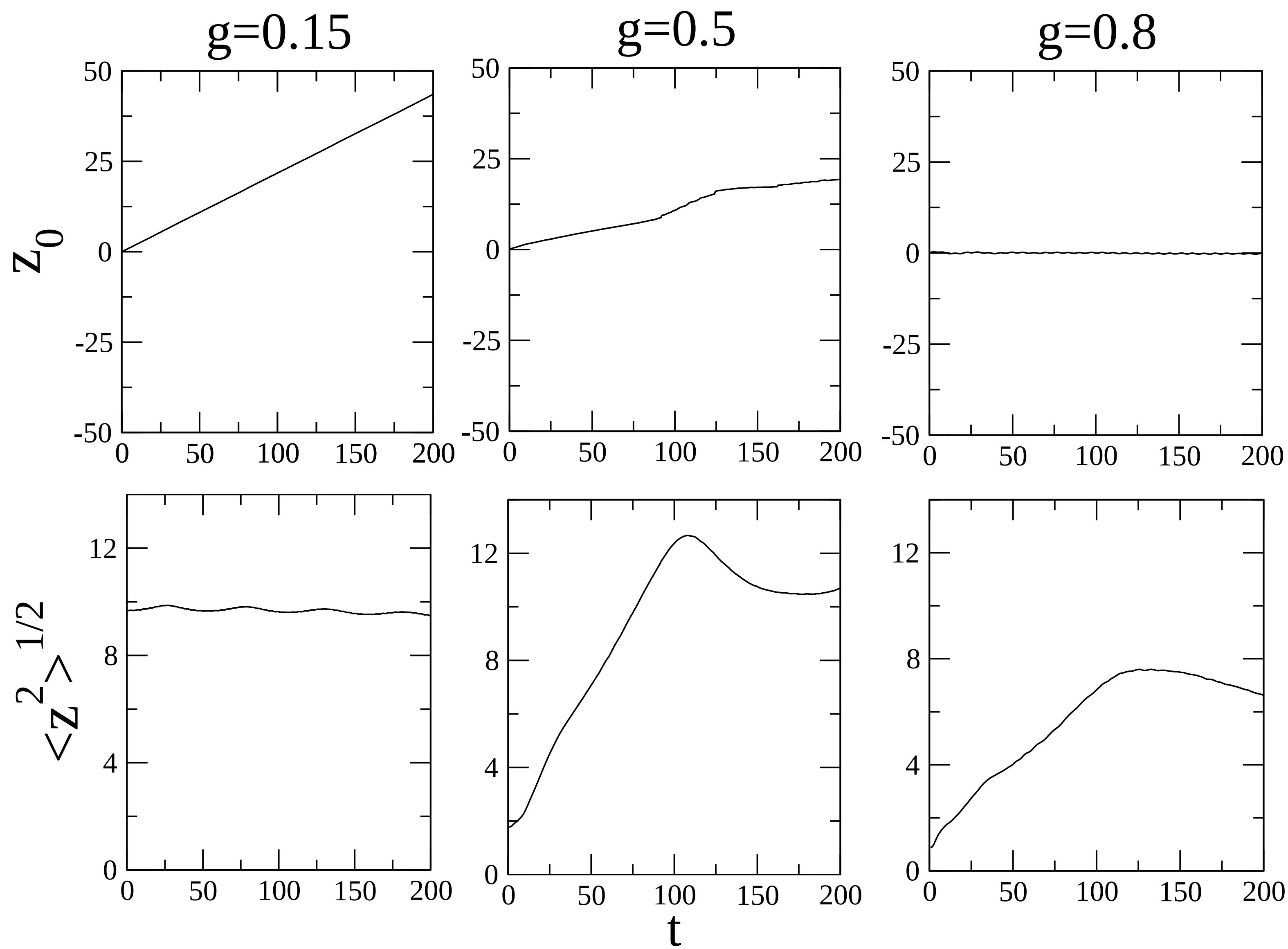

FIG. 4: Center of mass z_0 and average axial width $\langle z^2\rangle^{1/2}$ of the soliton as functions of time. Initial momentum is $p = 0.25$. Parameters of the optical lattice: $k_L = 1$ and $V_0 = 0.5$.

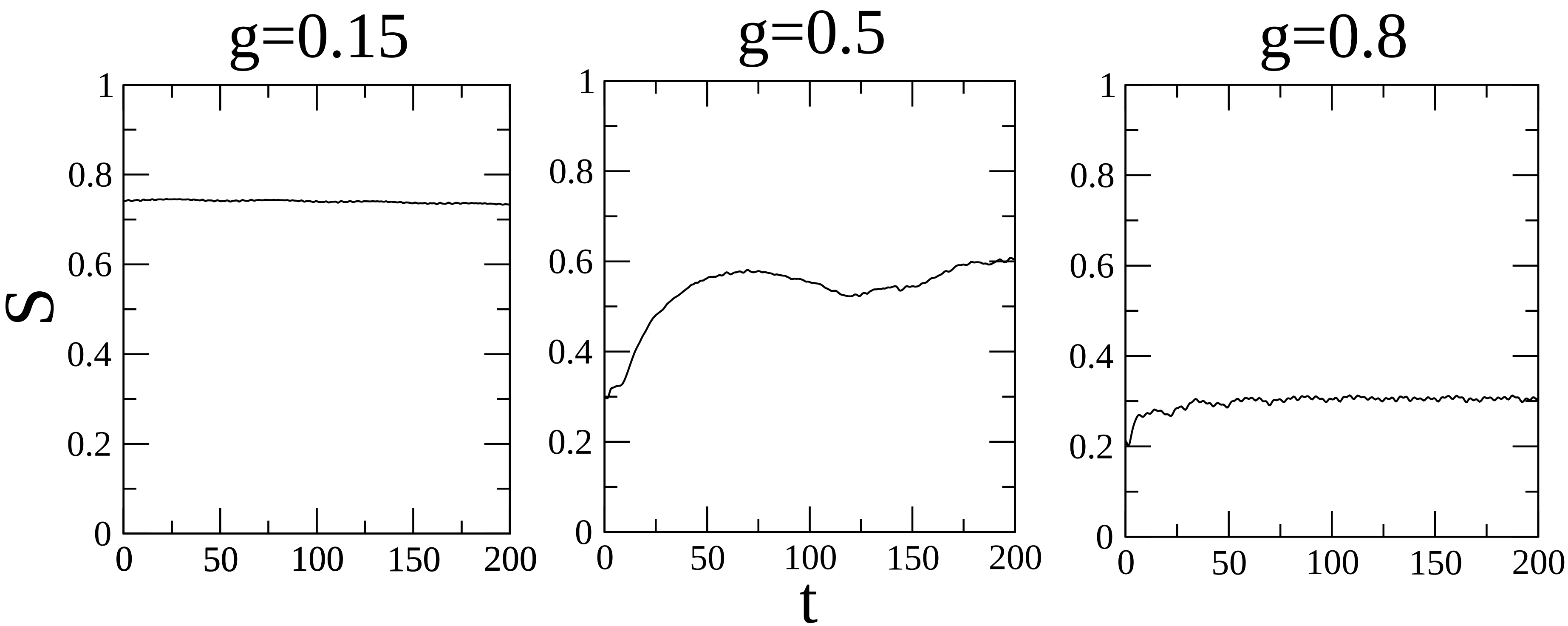

FIG. 5: Effective entropy $S(t)$ as a function of time t. Initial momentum is $p = 0.25$. Parameters of the optical lattice: $k_L = 1$ and $V_0 = 0.5$.

Our numerical simulations ($136 \times 2 = 272$ runs!) reveal the existence of three different dynamical regimes:

(i) **stable breathers**, i.e., solitons steadily moving at an almost constant velocity, with small-amplitude shape oscillations; for these steadily traveling solitons, the effective entropy $S(t)$ varies by $\leq 10\%$ in the course of the long evolution;

(ii) **dispersive dynamics**, in which case the soliton strongly spreads out in the course of the evolution; here solitons move at a variable speed, and their effective entropy increases by more than 10% against the initial value.

(iii) **localization**, in which a narrow soliton remains trapped in one lattice cell; here the solitions may be slightly dispersive at the initial stage of the evolution, but their effective entropy is always much smaller than in the other two cases.

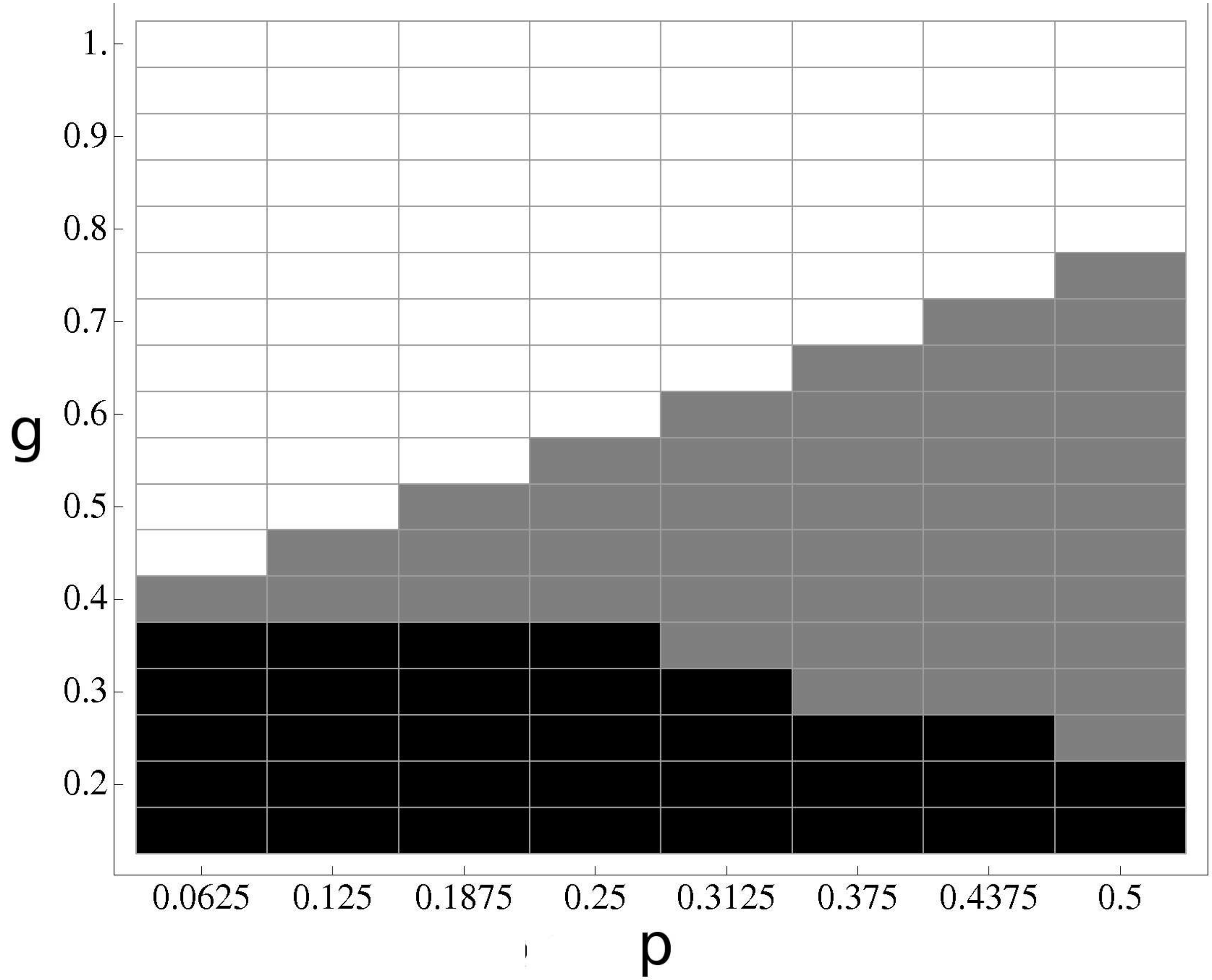

FIG. 6: Dynamical regimes in the plane of (p, g). Black region: steadily moving breather-like solitons; gray region: spreading out of the irregularly moving soliton; white region: localization (the center of mass does not move). Parameters of the optical lattice are $k_L = 1$ and $V_0 = 0.5$.

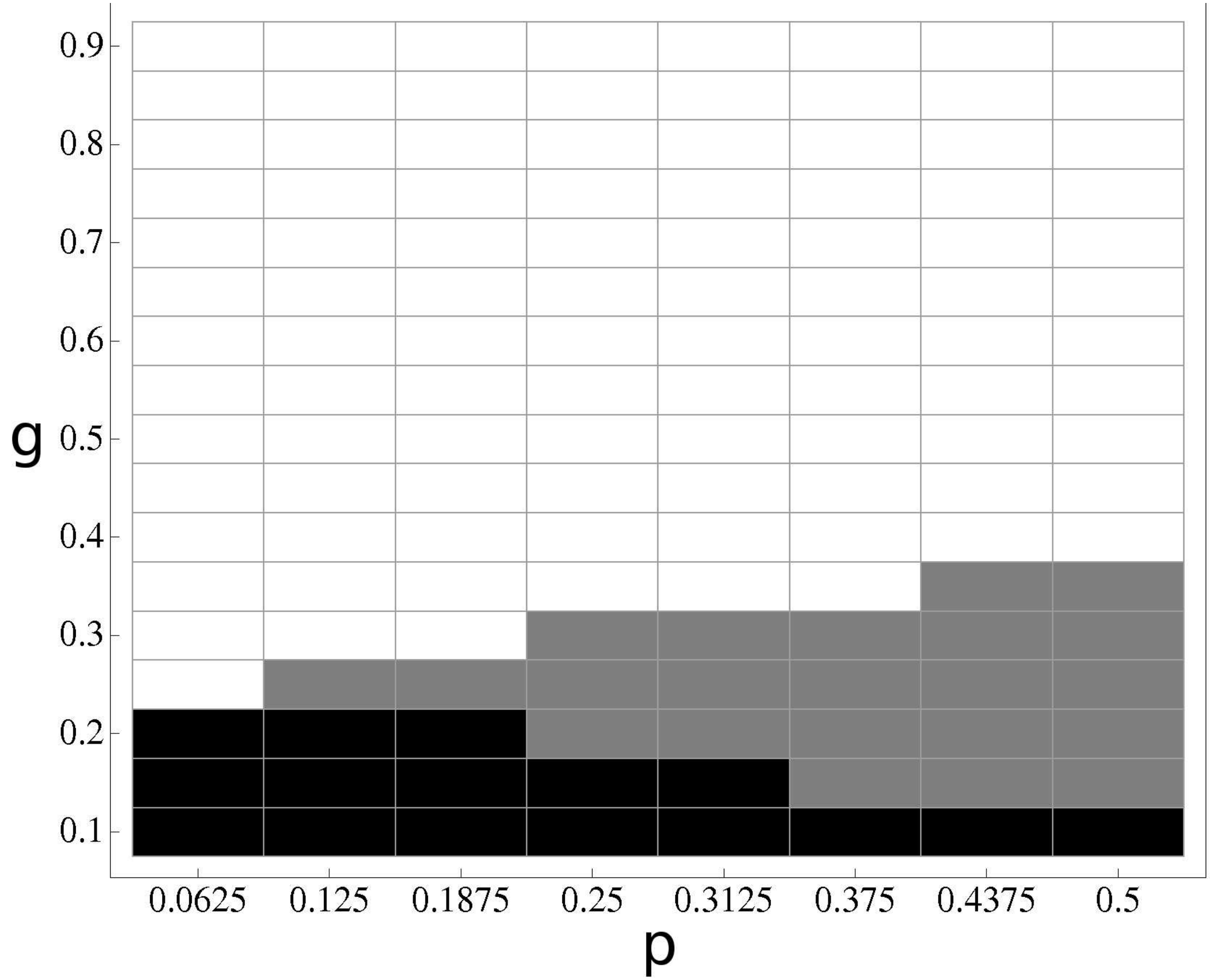

FIG. 7: Dynamical regimes in the plane of (p, g). Black region: steadily moving breather-like solitons; gray region: spreading out of the irregularly moving soliton; white region: localization (the center of mass does not move). Parameters of the optical lattice are $k_L = 1$ and $V_0 = 1$.

Conclusions

- NPSE is very reliable to describe confined attractive BEC in an axial optical lattice

- The ground-state bright soliton can occupy one or many-sites depending on inter-atomic stregth and lattice parameters

- Also the collapse depends on inter-atomic stregth and lattice parameters

- The behavior of a kicked bight soliton shows three different regimes:
  - breather-like;
  - irregular dynamics;
  - localization.

THANKS!!