Superconductivity, Superfluidity, and Bose-Einstein condensation

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Summary

- Bosons and fermions
- Laser Light
- Superconductivity
- Superfluidity
- Bose-Einstein condensation with ultracold atoms
- 2D systems: Kosterlitz-Thouless transition
- Conclusions

Bosons and fermions (I)

Any particle has an intrinsic angular momentum, called spin $\vec{S} = (S_x, S_y, S_z)$, characterized by two quantum numbers s ed m_s, where for s fixed one has $m_s = -s, -s+1, ..., s-1, s$, and in addition

$$S_z = m_s \hbar \; ,$$

with $\hbar$ ($1.054 \cdot 10^{-34}$ Joule$\times$seconds) the reduced Planck constant.
All the particles can be divided into two groups:
– **bosons**, characterized by an integer s:

$$s = 0, 1, 2, 3, ...$$

– **fermions**, characterized by a half-integer s:

$$s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, ...$$

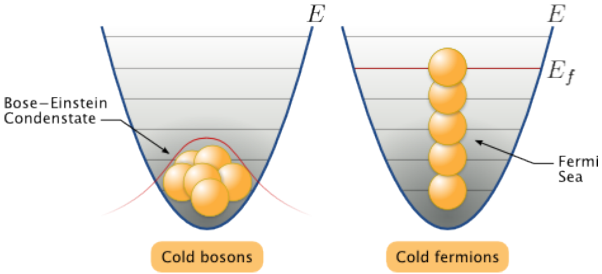
Examples: the photon is a boson ($s = 1$, $m_s = -1, 1$), while the electron is a fermion ($s = \frac{1}{2}$, $m_s = -\frac{1}{2}, \frac{1}{2}$).
Among "not elementary particles": helium $^4_2$He is a boson ($s = 0$, $m_s = 0$), while helium $^3_2$He is a fermion ($s = \frac{1}{2}$, $m_s = -\frac{1}{2}, \frac{1}{2}$).

Bosons and fermions (II)

A fundamental experimental result: identical bosons and identical fermions have a very different behavior!!

– Identical bosons can occupy the same single-particle quantum state, i.e. thay can stay together; if all bosons are in the same single-particle quantum state one has Bose-Einstein condensation.

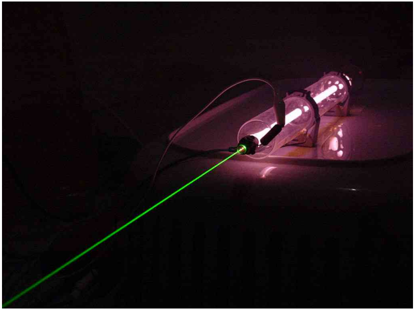
– Identical fermions CANNOT occupy the same single-particle quantum state, i.e. they somehow repel each other: Pauli exclusion principle.



Identical bosons (a) and identical spin-polarized fermions (b) in a harmonic trap at very low temperature.

Laser light (I)

In 1955 the first laser light devices were obtained by Charles Townes, Nikolay Basov, and Aleksandr Prokhorov.



The laser light is an example of Bose-Einstein condensation (out of thermal equilibrium): the photons (zero mass and spin 1) of the laser beam have

i) the same energy: monochromaticity

ii) the same linear momentum: unidirectionality

iii) the same phase: coherence.

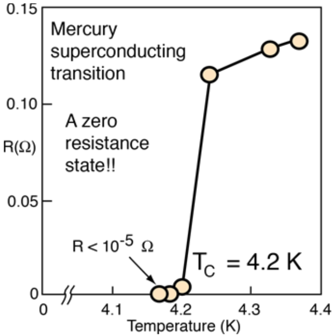
Laser light (II)

LASER = Light Amplification by Stimulated Emission of Radiation.

Laser light is now used for several applications:
– **material processing** (drilling, cutting, welding);
– **iindustrial and civil measurements** (laser interferometers for metrology, wire diameters, granulometers, roughness meters, relief systems of deformation fields);
– **telecommunications and optical fibers**;
– **medicine** (ophthalmology and general surgery).

Superconductivity (I)

Superconductivity is a phenomenon of exactly zero electrical resistance and expulsion of magnetic flux fields occurring in certain materials when cooled below a characteristic critical temperature T_c.
It was discovered in 1911 by **Heike Kamerlingh Onnes**.



In 1957 **John Bardeen**, **Leon Cooper** and **Robert Schrieffer** suggested that in superconductivity, due to the ionic lattice, pairs of electrons behave like bosons, as somehow anticipated in 1950 by **Lev Landau** and **Vitaly Ginzburg**.

Superconductivity (II)

Critical temperature T_c of some superconductors at atmospheric pressure.

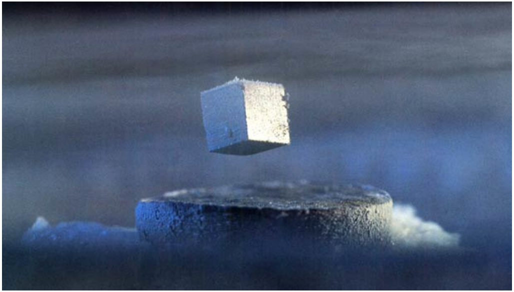
Material	**Symbol**	T_c **(Kelvin)**
Aluminium	$^{27}_{13}$Al	1.19
Tin	$^{120}_{50}$Sn	3.72
Mercury	$^{202}_{80}$Hg	4.16
Lead	$^{208}_{82}$Pb	7.20
Neodymium	$^{142}_{60}$Nb	9.30

In 1986 **Karl Alex Müller** and **Johannes Georg Bednorz** discovered high-T_c superconductors. These ceramic materials (cuprates) can reach the critical temperature of 133 Kelvin.

For these high-T_c superconductors the mechanisms which give rise to **pairing of electrons** are not fully understood.

Superconductivity (III)

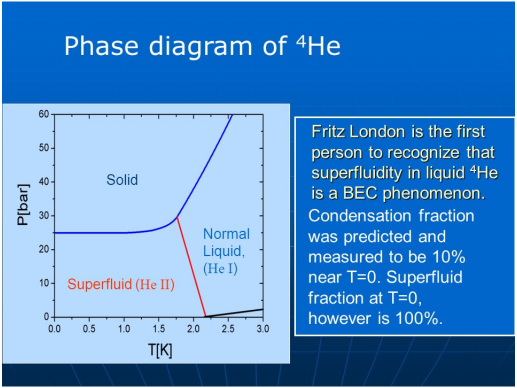
Superconductors have interesting properties. For instance the levitation of a magnetic material over a supercondutor (Meissner effect).



Some technological applications of superconductors:
– MAGLEV trains, based on magnetic levitation (mag-lev);
– SQUIDS, devices which measure extremely week mgnetic fields;
– very high magnetic fields for Magnetic Resonance in hospitals.

Superfluidity (I)

Superfluidity is the characteristic property of a fluid with zero viscosity, which therefore flows without loss of kinetic energy.



Superfluidity was discovered in 1937 by **Pyotr Kapitza**, who found that, at atmospheric pressure, below $T_\lambda = 2.16$ Kelvin helium 4 (^{4}He) not only remains liquid but it also shows zero viscosity. Moreover at T_λ the specific heat diverges.

Superfluidity (II)

A **fluid** can be described by the Navier-Stokes equations of hydrodynamics

$$\begin{aligned} \frac{\partial}{\partial t} n + \nabla \cdot (n\mathbf{v}) &= 0 \ , \\ m \frac{\partial}{\partial t}\mathbf{v} - \eta \ \nabla^2 \mathbf{v} + \nabla \left[\frac{1}{2} m v^2 + U_{ext} + \mu(n) \right] &= m \ \mathbf{v} \wedge (\nabla \wedge \mathbf{v}) \ , \end{aligned}$$

where $n(\mathbf{r}, t)$ is the density field and $\mathbf{v}(\mathbf{r}, t)$ is the velocity field. Here η is the viscosity, $U_{ext}(\mathbf{r})$ is the external potential acting on the particles of the fluid, and $\mu(n)$ is the equation of state of the fluid.

A superfluid is characterized by zero viscosity, i.e. $\eta = 0$, and irrotationality, i.e. $\nabla \wedge \mathbf{v} = \mathbf{0}$.

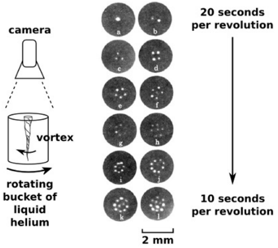
The equations of superfluid hydrodynamics are then

$$\begin{aligned} \frac{\partial}{\partial t} n_s + \nabla \cdot (n_s \mathbf{v}_s) &= 0 \ , \\ m \frac{\partial}{\partial t}\mathbf{v}_s + \nabla \left[\frac{1}{2} m v_s^2 + U_{ext} + \mu(n_s) \right] &= \mathbf{0} \ . \end{aligned}$$

They describe extremely well the superfluid ^{4}He, ultracold gases of alkali-metal atoms, and also several properties of superconductors.

Superfluidity (III)

In the 1950s **Lars Onsager**, **Richard Feynman**, and **Alexei Abrikosov** suggested that superconductors and superfluids can have quantized vortices.



Vortex line: superfluid density n_s and superfluid velocity v_s as a function of the cylindric radial coordinate R.

$$n_s(R) \simeq n_s(\infty)\left(1 - \frac{1}{1+\frac{R^2}{\xi^2}}\right) \quad \text{and} \quad v_s(R) = \frac{\hbar}{m}\frac{q}{R^2}$$

Quantized vortices have been **experimentally observed** in superconductors, superfluids and also laser light.

Superfludity (IV)

Quantized vortices can be explained by assuming that the dynamics of **superconductors** and **superfluids** is driven by a complex scalar field

$$\psi(\mathbf{r}, t) = |\psi(\mathbf{r}, t)| \; e^{i\theta(\mathbf{r},t)} \ ,$$

which satisfies the nonlinear Schrödinger equation (NLSE)

$$i\hbar \frac{\partial}{\partial t}\psi = \left[-\frac{\hbar^2}{2m}\nabla^2 + U_{ext} \right] \psi + \mu(|\psi|^2) \; \psi$$

and it is such that

$$n_s(\mathbf{r}, t) = |\psi(\mathbf{r}, t)|^2 \ , \qquad \mathbf{v}_s(\mathbf{r}, t) = \frac{\hbar}{m} \boldsymbol{\nabla}\theta(\mathbf{r}, t) \ .$$

In fact, under these assumptions, NLSE is practically equivalent to equations of superfluid hydrodynamics, and the multi-valued angle variable $\theta(\mathbf{r}, t)$ is such that

$$\oint_{\mathcal{C}} d\theta = \oint_{\mathcal{C}} \boldsymbol{\nabla}\theta(\mathbf{r}, t) \cdot d\mathbf{r} = 2\pi q \ ,$$

with $q = 0, \pm 1, \pm 2, \ldots$. The quantized vortex line is then obtained with

$$\theta(x, y, z) = q \arctan\left(\frac{y}{x}\right) \ .$$

Superfluidity (V)

The NLSE of the complex scalar field $\psi(\mathbf{r}, t)$ of superfluids

$$i\hbar \frac{\partial}{\partial t}\psi = \left[-\frac{\hbar^2}{2m}\nabla^2 + U_{ext} \right] \psi + \mu(|\psi|^2)\ \psi$$

admits the constant of motion (energy of the system)

$$H = \int \left\{ \frac{\hbar^2}{2m}|\nabla\psi|^2 + U_{ext}\ |\psi|^2 + \mathcal{E}(|\psi|^2) \right\}\ d^D\mathbf{r}\ ,$$

where $\mu(|\psi|^2) = \frac{\partial \mathcal{E}(|\psi|^2)}{\partial |\psi|^2}$. Taking into account that

$$\psi(\mathbf{r}, t) = n_s(\mathbf{r}, t)^{1/2}\ e^{i\theta(\mathbf{r},t)} \quad \text{with} \quad \mathbf{v}_s(\mathbf{r}, t) = \frac{\hbar}{m}\boldsymbol{\nabla}\theta(\mathbf{r}, t)$$

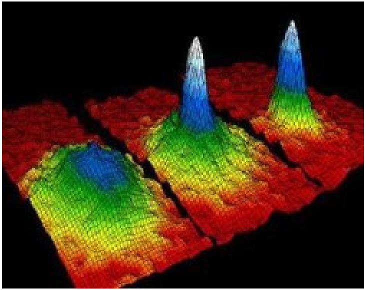
one finds

$$\frac{\hbar^2}{2m}|\nabla\psi|^2 = \frac{\hbar^2}{2m}\ n_s\ (\nabla\theta)^2 + \frac{\hbar^2}{8m}\frac{(\nabla n_s)^2}{n_s}\ ,$$

which are phase-stiffness energy and quantum pressure.

Bose-Einstein condensation with ultracold atoms (I)

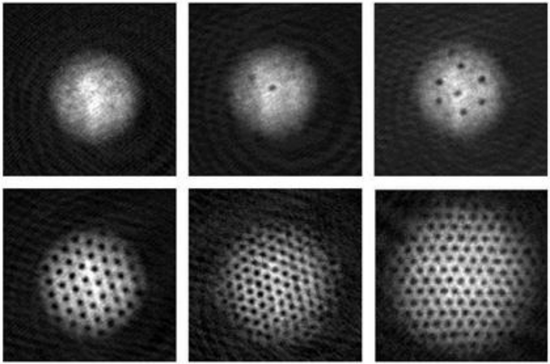
In 1995 Eric Cornell, Carl Wieman e Wolfgang Ketterle achieved Bose-Einstein condensation (BEC) cooling gased of ^{87}Rb and ^{23}Na. For these bosonic systems, which are very dilute and ultracold, the critical temperature to reach the BEC is about $T_c \simeq 100$ nanoKelvin.



Density profiles of a gas of Rubidium atoms: formation of the Bose-Einstein condensate. For an atom of ^{87}Rb the total nuclear spin is $I = \frac{3}{2}$, the total electronic spin is $S = \frac{1}{2}$, and the total atomic spin is $F = 1$ o $F = 2$: the neutral ^{87}Rb atom is a boson.

Bose-Einstein condensation with ultracold atoms (II)

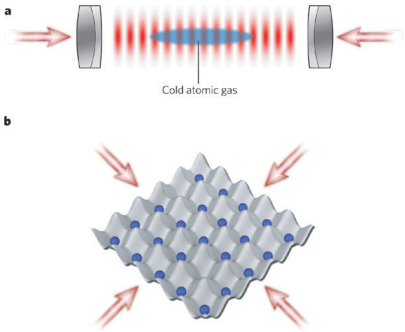
An interesting consequence of Bose-Einstein condensation with ultracold atoms is the possibility to generate quantized vortices.



Formation of quantized vortices in a condensed gas of ^{87}Rb atoms. The number of vortices increases by increasing the rotational frequency of the system.

Bose-Einstein condensation with ultracold atoms (III)

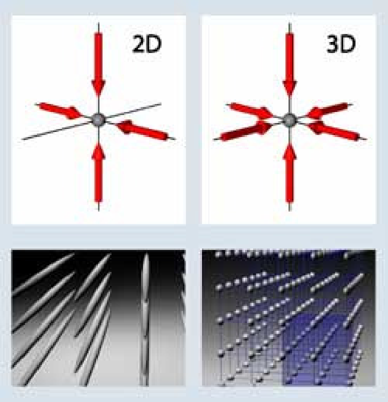
Nel 2002 the group of Immanuel Block at Munich obtained, with the intereference of counterpropagating laser beams inside an optical cavity delimitated by mirrors, a stationary optical lattice. Quite remarkably, this optical lattice traps ultracold atoms.



Due to the coupling between the electric field $\mathbf{E}(\mathbf{r}, t)$ of the laser beams and dipolar electric moment $\mathbf{d}$ of each atom, the resulting periodic potential traps neutral atoms in the minuma of the the optical lattice.

Bose-Einstein condensation with ultracold atoms (IV)

In the last years the studies of atoms trapped with light (atoms in optical lattices) have been refined.



Tuning the intensity and the shape of the optical lattice it is now possible to trap atoms in very different configurations, and also very different numbers of atoms: from many atoms per site to a single atom per site.

2D systems: Kosterlitz-Thouless transition (I)

In three spatial dimensions ($D = 3$), the complex scalar field of superfluids is called order parameter of the system and it is often identified as the **macroscopic wavefunction** of Bose-Einstein condensation (BEC), where a macroscopic fraction of particles occupies the same single-particle quantum state.

BEC phase transition: For an ideal gas of non-interacting identical bosons there is BEC only below a critical temperature T_{BEC}. In particular one finds

$$k_B\, T_{BEC} = \begin{cases} \frac{1}{2\pi\zeta(3/2)^{2/3}} \frac{\hbar^2}{m} n^{2/3} & \text{for } D = 3 \\ 0 & \text{for } D = 2 \\ \text{no solution} & \text{for } D = 1 \end{cases}$$

where D is the spatial dimension of the system, n is the number density, and $\zeta(x)$ is the Riemann zeta function.

This result due to Einstein (1925), which says that there is no BEC at finite temperature for $D \leq 2$ in the case of non-interacting bosons, was extended to interacting systems by **David Mermin** and **Herbert Wagner** in 1966.

2D systems: Kosterlitz-Thouless transition (II)

Despite the absence of BEC in 2D, in 1972 **Kosterlitz** and **Thouless** (but also **Vadim Berezinskii** (1935-1980)) suggested that a 2D fluid can be superfluid below a critical temperature, the so-called Berezinskii-Kosterlitz-Thouless critical temperature T_{BKT}.
They analyzed the 2D XY model, which was originally used to describe the magnetization in a planar lattice of classical spins. The Hamiltonian of the continuous 2D XY model is given by

$$H = \int \frac{J}{2} (\nabla\theta)^2 \; d^2\mathbf{r} \; ,$$

where $\theta(\mathbf{r})$ is the angular field and J is the phase stiffness (rigidity).
This Hamiltonian is nothing else than the constant of motion (energy)

$$H = \int \left\{ \frac{\hbar^2}{2m} |\nabla\psi|^2 + \mathcal{E}(|\psi|^2) \right\} \; d^2\mathbf{r}$$

of the 2D NLSE of the complex scalar field $\psi(\mathbf{r}, t)$ of 2D superfluids.
Here $U_{ext} = 0$ and

$$\psi(\mathbf{r}) = n_s^{1/2} \; e^{i\theta(\mathbf{r})}$$

with $J = n_s(\hbar^2/m)$, and neglecting the bulk energy $\mathcal{E}(n_s)L^2$.

2D systems: Kosterlitz-Thouless transition (III)

Formally, one can rewrite the angular field as follows

$$\theta(\mathbf{r}) = \theta_0(\mathbf{r}) + \theta_v(\mathbf{r}) ,$$

where $\theta_0(\mathbf{r})$ has zero circulation (no vortices) while $\theta_v(\mathbf{r})$ encodes the contribution of quantized vortices.

After some manipulations, the 2D XY Hamiltonian can be rewritten as

$$H = \int \frac{J}{2}(\nabla\theta_0)^2 d^2\mathbf{r} + \sum_{i\neq j} V(\mathbf{r}_i - \mathbf{r}_j) q_i q_j - \sum_j \mu_c \, q_j^2$$

where the second term describes quantized vortices located at position $\mathbf{r}_i$ with quantum numbers q_i, interacting through a 2D Coulomb-like potential

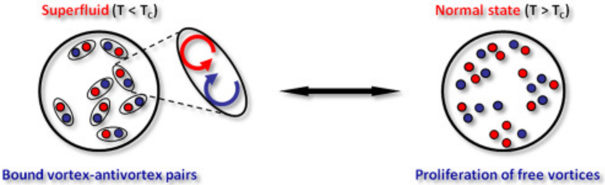
$$V(r) = -2\pi J \ln\left(\frac{r}{\xi}\right) ,$$

where ξ is healing length, i.e. the cutoff length defining the vortex core size, and μ_c the energy associated to the creation of a vortex.

2D systems: Kosterlitz-Thouless transition (IV)

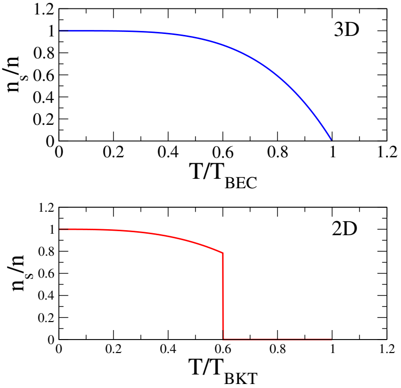
The analysis of **Kosterlitz** and **Thouless** on this 2D Coulomb-like gas of quantized vortices has shown that:

- As the temperature T increases vortices start to appear in vortex-antivortex pairs (mainly with $q = \pm 1$).
- The pairs are bound at low temperature until at the critical temperature $T_c = T_{BKT}$ an unbinding transition occurs above which a proliferation of free vortices and antivortices is predicted.
- The phase stiffness J and the vortex energy μ_c are renormalized.
- The renormalized superfluid density $n_s = J(m/\hbar^2)$ decreases by increasing the temperature T and jumps to zero above $T_c = T_{BKT}$.



2D systems: Kosterlitz-Thouless transition (V)

An important prediction of the Kosterlitz-Thouless transition is that, contrary to the 3D case, in 2D the superfluid fraction n_s/n jumps to zero above a critical temperature.



For 3D superfluids the transition to the normal state is a BEC phase transition, while in 2D superfluids the transition to the normal state is something different: a topological phase transition.

Conclusions

At the end of this talk I hope you may comment using the words of Enrico Fermi:

"Before I came here I was confused about this subject.
Having listened to your lecture I am still confused.
But on a higher level.

THANK YOU VERY MUCH FOR YOUR ATTENTION!

Slides online: http://materia.dfa.unipd.it/salasnich/talk-patna19.pdf