Dynamics of fermions and Fermi-Bose mixture in a double-well potential

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Ourense, April 14, 2010

Summary

- Spin-polarized fermions in a double-well
- Exact tunneling dynamics of ideal fermions
- Fermions interacting with a localized BEC
- Conclusions

Spin-polarized fermions in double-well

We consider a confined dilute and ultracold spin-polarized gas of N_F fermionic atoms of mass m in a double-well potential.* In practice: an ideal Fermi gas because the s-wave interaction of two particle with the same spin is forbidden by Fermi statistics.

The trapping potential of the system is given by

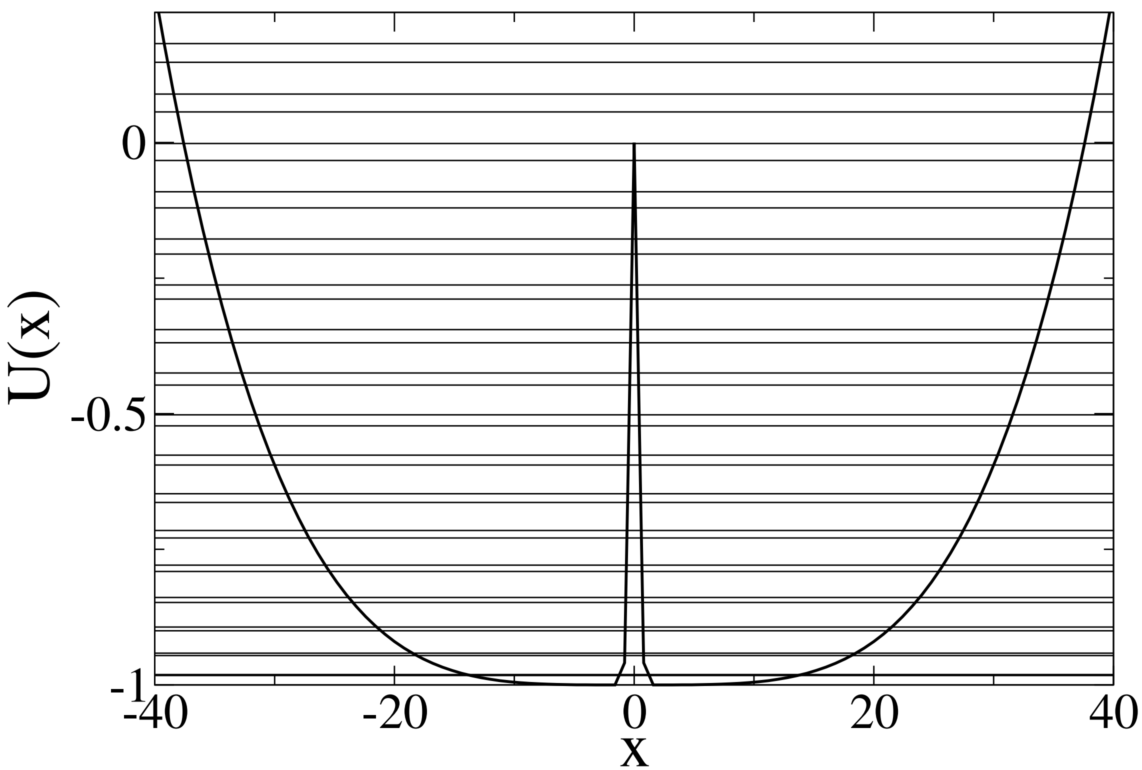
$$V(\mathbf{r}) = U(x) + \frac{1}{2} m\omega_\perp^2 (y^2 + z^2) \; , \tag{1}$$

We suppose that the system is one-dimensional (1D), due to a strong radial transverse harmonic confinement, with a symmetric double-well trapping potential in the longitudinal axial direction, that we will denote by $U(x)$. Thus, the transverse energy $\hbar\omega_\perp$ is much larger than the characteristic energy of fermions in the axial direction.

*A similar system has beeen studied by S. Zöllner *et al.*, PRL **100**, 040401 (2008); K. Ziegler, PRA **81**, 034701 (2010).

We model the axial double-well trapping potential $U(x)$ as

$$U(x) = Ax^4 + B(e^{-Cx^2} - 1) \; . \tag{2}$$



The double well potential for $A = 5 \cdot 10^{-7}$, $B = 1$, and $C = 5$. For these values there are 30 energetic levels (corresponding to 15 doublets) with energy smaller than the height of the barrier. Energies in units of $\hbar\omega_\perp$, lengths in units of $a_\perp = \sqrt{\hbar/m\omega_\perp}$. [LS *et al.*, PRA **81**, 023614 (2010)]

The single-particle stationary Schrödinger equation of the 1D problem can be written as

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x)\right]\phi_{\alpha,j}(x) = \epsilon_{\alpha,j}\phi_{\alpha,j}(x) \ , \tag{3}$$

where $\phi_{\alpha,j}(x)$ are the complete set of orthonormal eigenfunctions and $\epsilon_{\alpha,j}$ the corresponding eigenenergies. Here $\alpha = 1, 2, 3, ...$ gives the ordering number of the doublets of quasi-degenerate states, and $j = S, A$ gives the symmetry of the eigenfunctions (S means symmetric and A means anti-symmetric).

Due to the symmetry of the problem, it is useful to introduce a complete set of orthonormal functions

$$\xi_{\alpha,R}(x) = \frac{1}{\sqrt{2}}(\phi_{\alpha,S}(x) + \phi_{\alpha,A}(x)) \ , \tag{4}$$

and

$$\xi_{\alpha,L}(x) = \frac{1}{\sqrt{2}}(\phi_{\alpha,S}(x) - \phi_{\alpha,A}(x)) \ . \tag{5}$$

If we fix the phase of $\phi_{\alpha,S}$ and $\phi_{\alpha,A}$, such that $\phi'_{\alpha,j}(+\infty) < 0$ for both $j = S$ and $j = A$ then $\xi_{\alpha,L}$ and $\xi_{\alpha,R}$ are mainly localized in the left and right well respectively, at least for the low lying states.

The exact many-body Hamiltonian of the system can be written as

$$\hat{H} = \sum_{\alpha=1}^{\infty} \sum_{i=L,R} \bar{\epsilon}_\alpha \ \hat{c}^\dagger_{\alpha,i} \hat{c}_{\alpha,i} - \sum_{\alpha=1}^{\infty} J_\alpha (\hat{c}^\dagger_{\alpha,L} \hat{c}_{\alpha,R} + \hat{c}^\dagger_{\alpha,R} \hat{c}_{\alpha,L}) \ , \tag{6}$$

where $\hat{n}_{F,\alpha,i} = \hat{c}^\dagger_{\alpha,i} \hat{c}_{\alpha,i}$ is the fermionic number operator of the α-th state in the i well ($i = L, R$: L means left and R means right). Here we have

$$\bar{\epsilon}_\alpha = \frac{\epsilon_{\alpha,S} + \epsilon_{\alpha,A}}{2} \ , \tag{7}$$

while

$$J_\alpha = \frac{\epsilon_{\alpha,A} - \epsilon_{\alpha,S}}{2} \tag{8}$$

is the energy of hopping from the L to the R well within the same doublet and gives directly the Rabi linear frequency of the α-th doublet as

$$\nu_\alpha = \frac{J_\alpha}{\pi \hbar} \ . \tag{9}$$

Exact tunneling dynamics of ideal fermions

The time-dependent density profile $n_F(x,t)$ of the Fermi system can be written as

$$n_F(x,t) = \sum_{\alpha=1}^{\infty} \sum_{i=L,R} n_{F,\alpha,i}(t)\ \xi_{\alpha,i}(x)^2 \ , \tag{10}$$

where $n_{F,\alpha,i}(t) = \langle \hat{n}_{F,\alpha,i}(t) \rangle = \langle \hat{c}^{\dagger}_{\alpha,i} \hat{c}_{\alpha,i} \rangle$.

Here $\langle \cdots \rangle$ is the thermal average obtained with the "unperturbed" Hamiltonian ($J_\alpha = 0$), which implies the initial conditions

$$n_{F,\alpha,L}(0) = \frac{1}{e^{\beta\,(\bar{\epsilon}_\alpha - \mu_{F,L})} + 1} = f_{\alpha,L} \ , \tag{11}$$

$$n_{F,\alpha,R}(0) = \frac{1}{e^{\beta\,(\bar{\epsilon}_\alpha - \mu_{F,R})} + 1} = f_{\alpha,R} \ , \tag{12}$$

where the chemical potentials $\mu_{F,L}$ and $\mu_{F,R}$ are fixed by the number of particles in the left and right wells at the initial time ($t = 0$).

By using the Heisenberg equations of motions of the operators $\hat{c}_{\alpha,i}^{\dagger}$ and $\hat{c}_{\alpha,i}$ it is not difficult to show that

$$n_{F,\alpha,L}(t) = f_{\alpha,L} \ \cos^2(\pi\nu_\alpha t) + f_{\alpha,R} \ \sin^2(\pi\nu_\alpha t) \ , \tag{13}$$

$$n_{F,\alpha,R}(t) = f_{\alpha,R} \ \cos^2(\pi\nu_\alpha t) + f_{\alpha,L} \ \sin^2(\pi\nu_\alpha t) \ . \tag{14}$$

The population imbalance $z_\alpha(t)$ within the α-th double is

$$z_{F,\alpha}(t) = n_{F,\alpha,L}(t) - n_{F,\alpha,R}(t) = (f_{\alpha,L} - f_{\alpha,R}) \ \cos(2\pi\nu_\alpha t) \ , \tag{15}$$

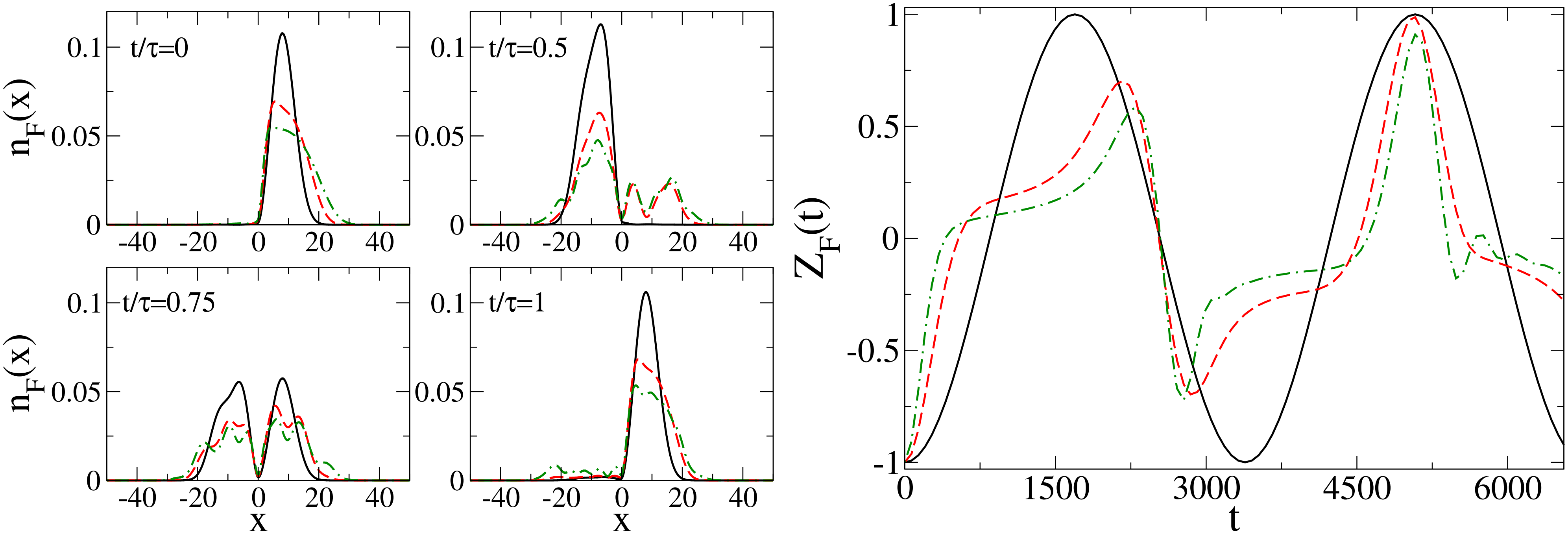
and the total fermionic imbalance $Z_F(t)$ is given by

$$Z_F(t) = \frac{1}{N_F}\sum_{\alpha=1}^{\infty} z_\alpha(t) = \frac{1}{N}\sum_{\alpha=1}^{\infty}(f_{\alpha,L} - f_{\alpha,R}) \ \cos(2\pi\nu_\alpha t) \ . \tag{16}$$

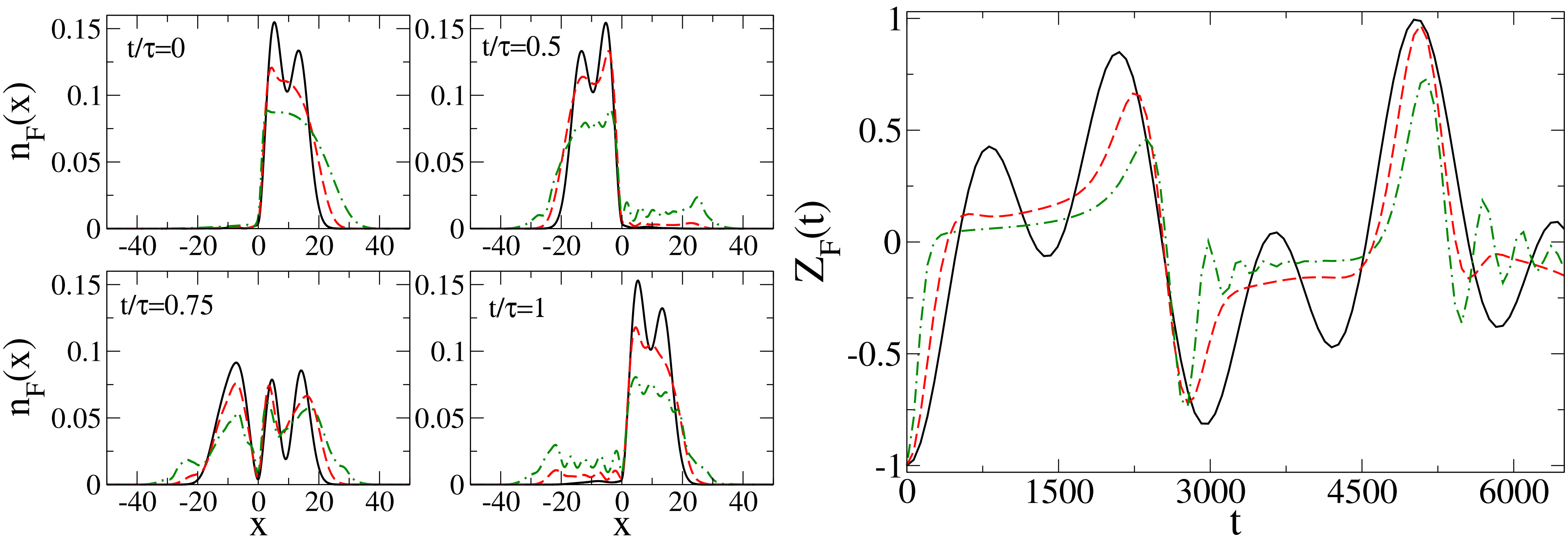
We consider ^{40}K atoms with $\omega_\perp = 160$ kHz, which implies $a_\perp \simeq 0.1$ μm. In addition, we set

$$f_{\alpha,L} = 0 \qquad \text{for any } \alpha \ , \tag{17}$$

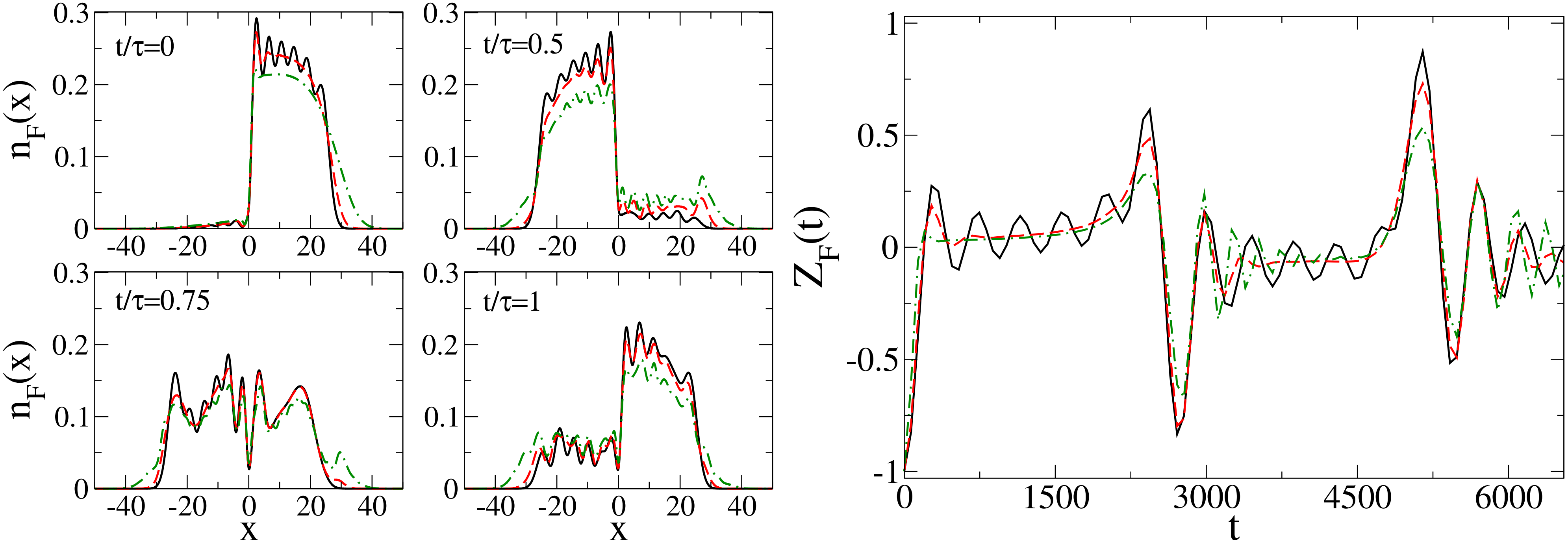
i.e. we suppose that initially all fermionic atoms are in the right well.



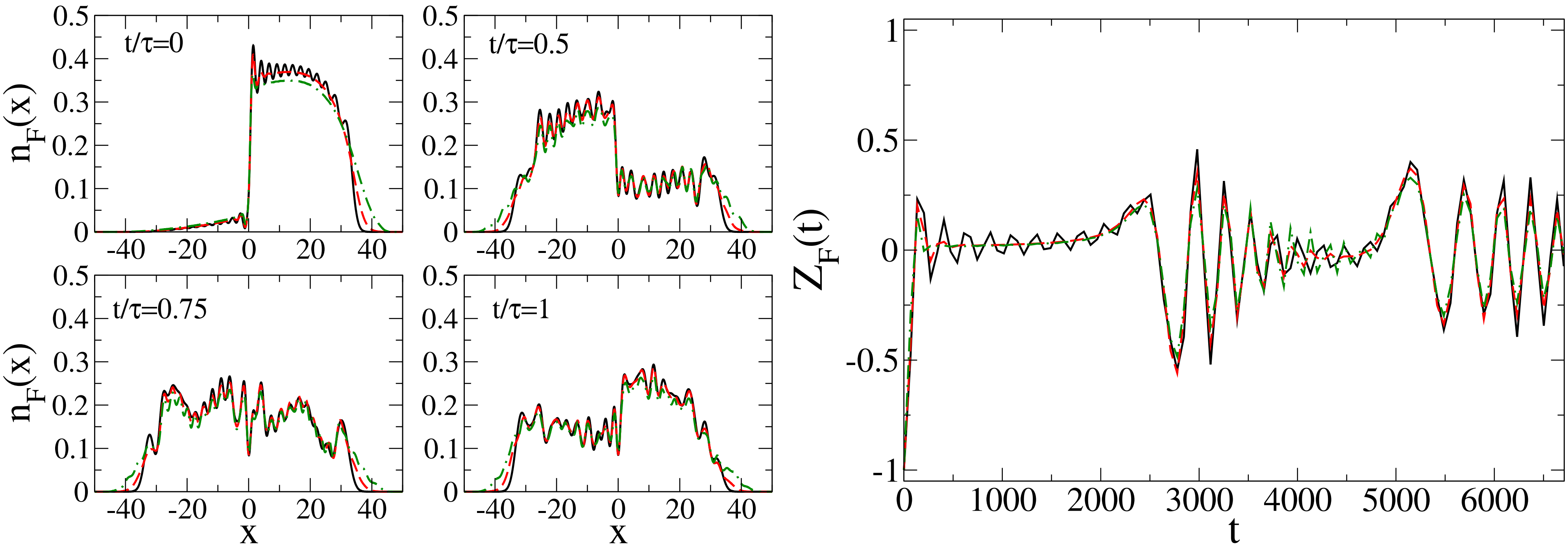
Density profile $n_F(x)$ and fermionic imbalance $Z_F(t)$ of the Fermi gas with $N_F = 1$. Solid lines: $k_B T/(\hbar\omega_\perp) = 0$; dashed lines: $k_B T/(\hbar\omega_\perp) = 0.05$; dot-dashed lines: $k_B T/(\hbar\omega_\perp) = 0.1$. Here $\tau = 1/\nu_1 = 3378\ \omega_\perp$ is the period related to the Rabi linear frequency ν_1 of the lowest doublet. $\omega_\perp$ is the angular frequency of the transverse harmonic confinement. [LS *et al.*, PRA **81**, 023614 (2010)]



Density profile $n_F(x)$ and fermionic imbalance $Z_F(t)$ of the Fermi gas with $N_F = 2$. Solid lines: $k_B T/(\hbar\omega_\perp) = 0$; dashed lines: $k_B T/(\hbar\omega_\perp) = 0.1$; dot-dashed lines: $k_B T/(\hbar\omega_\perp) = 0.2$. Here $\tau = 1/\nu_1 = 3378\ \omega_\perp$ is the period related to the Rabi linear frequency μ_1 of the lowest doublet. $\omega_\perp$ is the angular frequency of the transverse harmonic confinement. [LS *et al.*, PRA **81**, 023614 (2010)]



Density profile $n_F(x)$ and fermionic imbalance $Z_F(t)$ of the Fermi gas with $N_F = 6$. Solid lines: $k_B T/(\hbar\omega_\perp) = 0$; dashed lines: $k_B T/(\hbar\omega_\perp) = 0.1$; dot-dashed lines: $k_B T/(\hbar\omega_\perp) = 0.2$. Here $\tau = 1/\nu_1 = 3378\ \omega_\perp$ is the period related to the Rabi linear frequency ν_1 of the lowest doublet. $\omega_\perp$ is the angular frequency of the transverse harmonic confinement. [LS *et al.*, PRA **81**, 023614 (2010)]



Density profile $n_F(x)$ vs. fermionic imbalance $Z_F(t)$ of the Fermi gas with $N_F = 12$. Solid lines: $k_B T/(\hbar\omega_\perp) = 0$; dashed lines: $k_B T/(\hbar\omega_\perp) = 0.1$; dot-dashed lines: $k_B T/(\hbar\omega_\perp) = 0.3$. Here $\tau = 1/\nu_1 = 3378\ \omega_\perp$ is the period related to the Rabi linear frequency ν_1 of the lowest doublet. $\omega_\perp$ is the angular frequency of the transverse harmonic confinement. [LS *et al.*, PRA **81**, 023614 (2010)]

Fermions interacting with a localized BEC

We consider now a spin-polarized fermionic gas in interaction with a Bose-Einstein condensate (BEC).* The dynamics of the mixture can be described by the following set of coupled equations

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x) + g_B N_B|\Psi|^2 + g_{BF} n_F\right]\Psi \ , \qquad (18)$$

$$i\hbar\frac{\partial\chi_j}{\partial t} = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x) + N_B g_{BF}|\Psi|^2\right]\chi_j \ , \qquad (19)$$

where $n_F(x,t) = \sum_{j=1}^{N_F} |\chi_j(x,t)|^2$ denotes the fermionic density with $\chi_j(x,t)$ the set of <u>orthonormal</u> wave functions which satisfy Eq. (19), $\Psi(x,t)$ is the bosonic wavefunction and such that $n_B(x,t) = N_B|\Psi(x,t)|^2$ is the bosonic density with N_B the total number of bosons.

*A similar system has been studied by S F Caballero-Bentez *et al.*, JPB **42**, 215308 (2009).

We choose $N_F = 10$, $N_B = 470$, $a_B/a_\perp = -0.001$, and $a_{BF}/a_\perp = 0.001$.

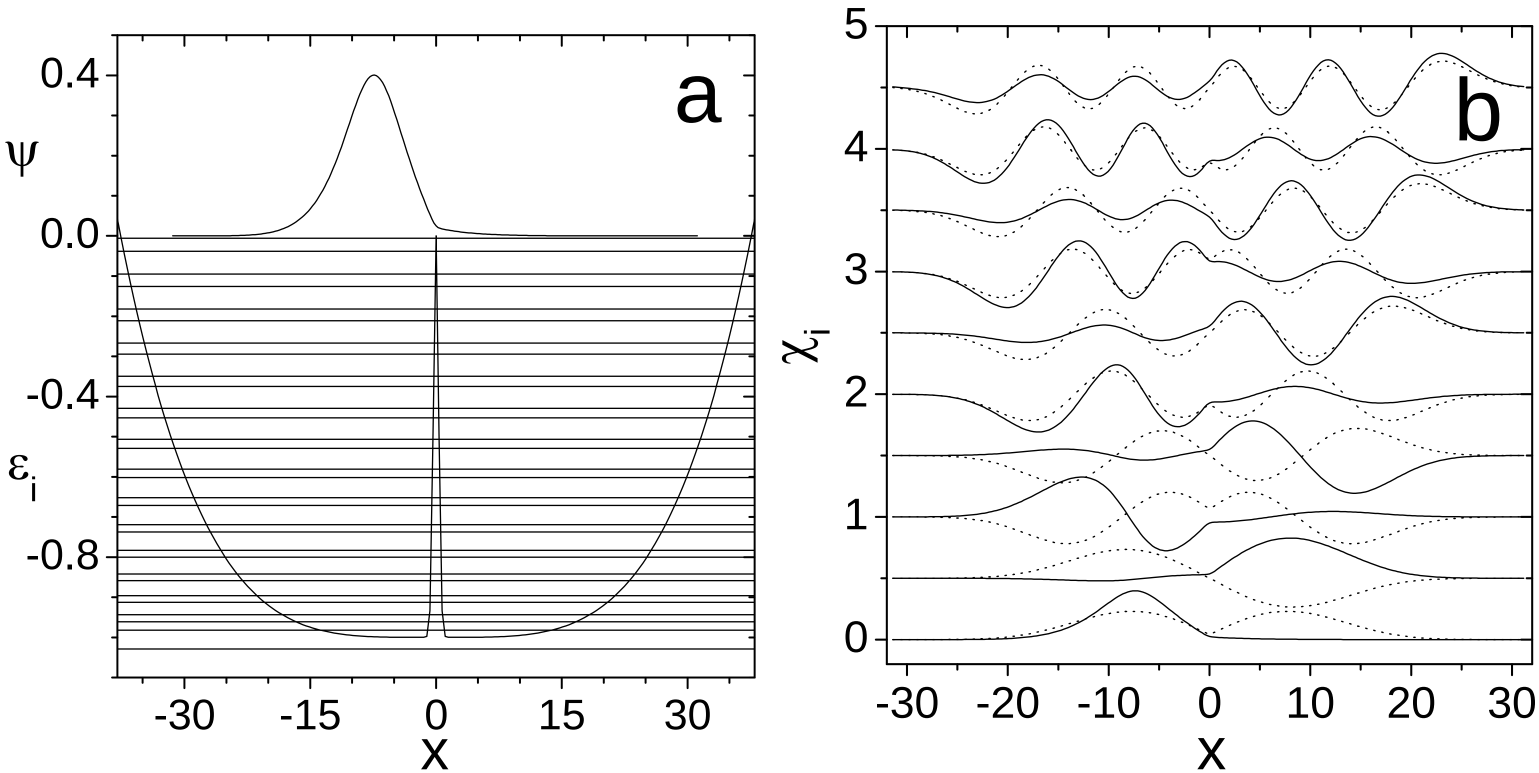
Under these conditions the bosonic cloud is self-trapped.*

The nonlinear Schrödinger equations for the fermionic single-particle wave functions $\chi_j(x)$ can be well approximated by the linear Schrödinger equations

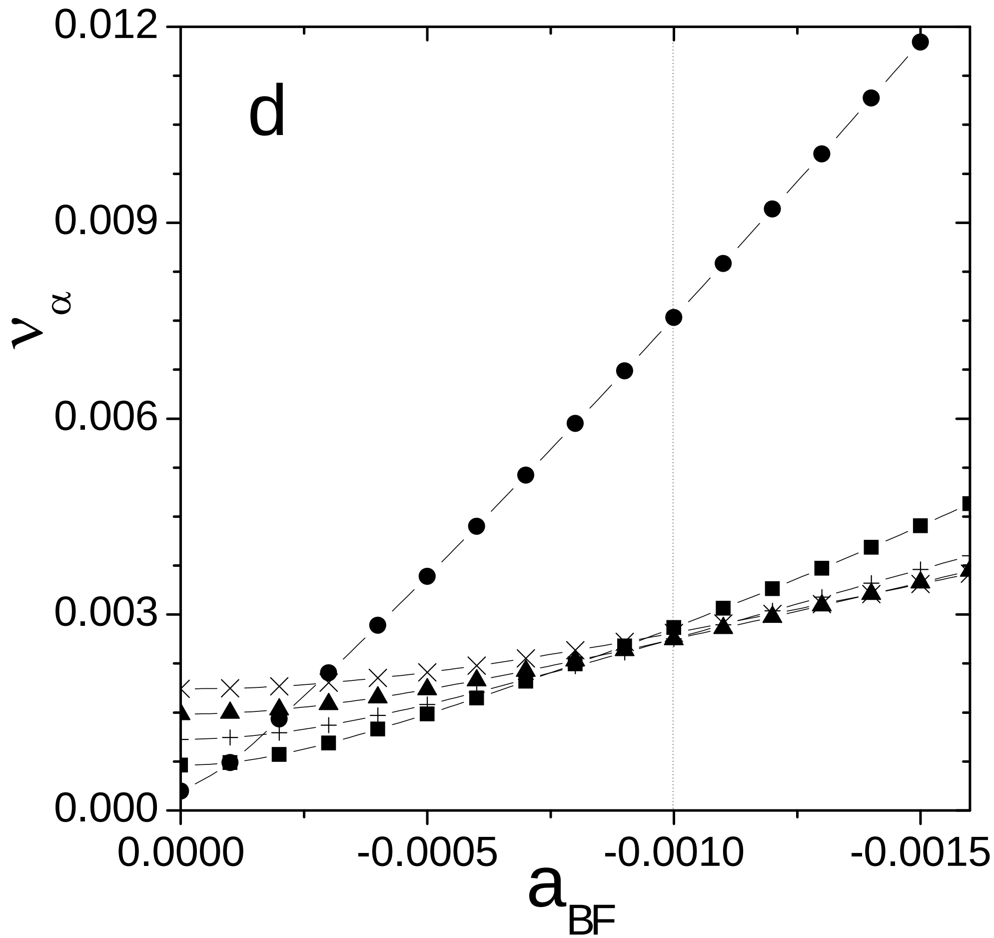
$$i\hbar\frac{\partial\chi_j}{\partial t} = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x) + g_{BF}\bar{n}_B(x)\right]\chi_j \, , \tag{20}$$

with $\bar{n}_B(x)$ denoting the stationary bosonic density. We have numerically verified that indeed the bosonic cloud is practically stationary.

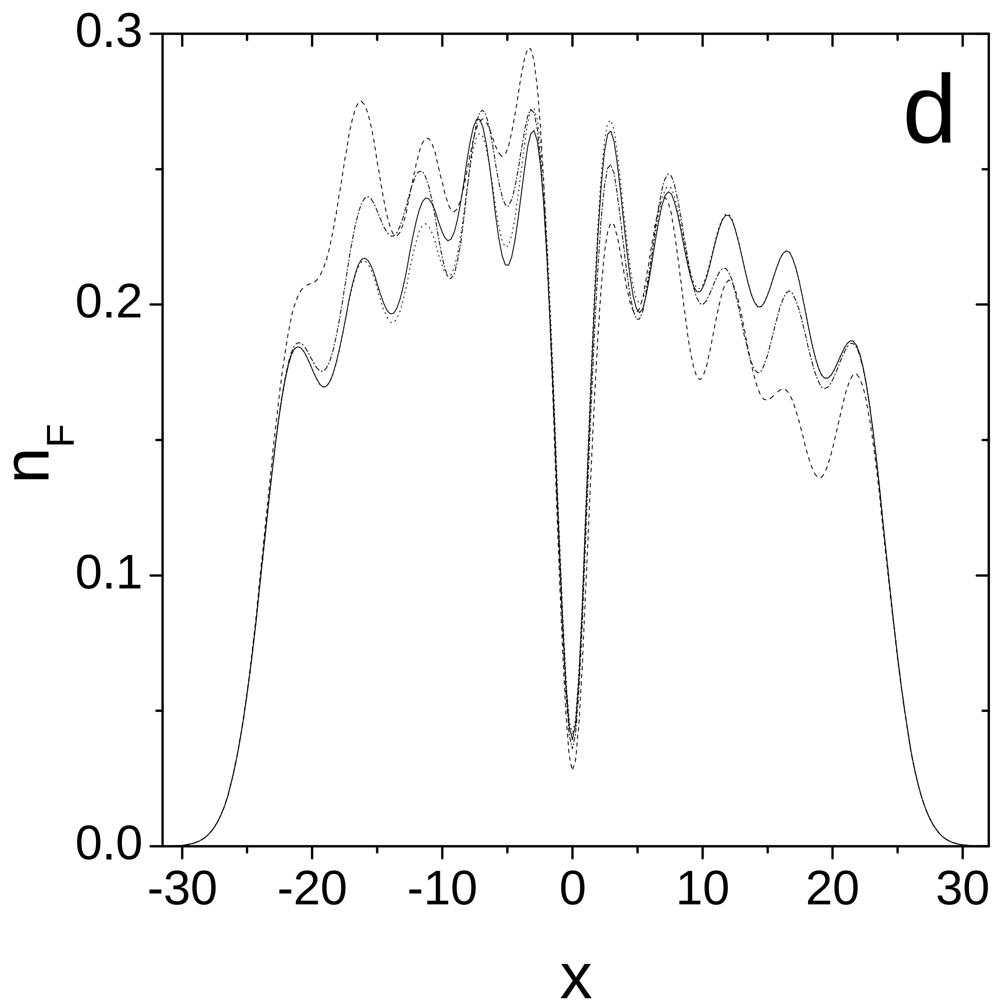
*The self-trapping condition for BEC is $|a_B|N_B/a_\perp > (\epsilon_{1,A} - \epsilon_{1,S})/(\hbar\omega_\perp)$.



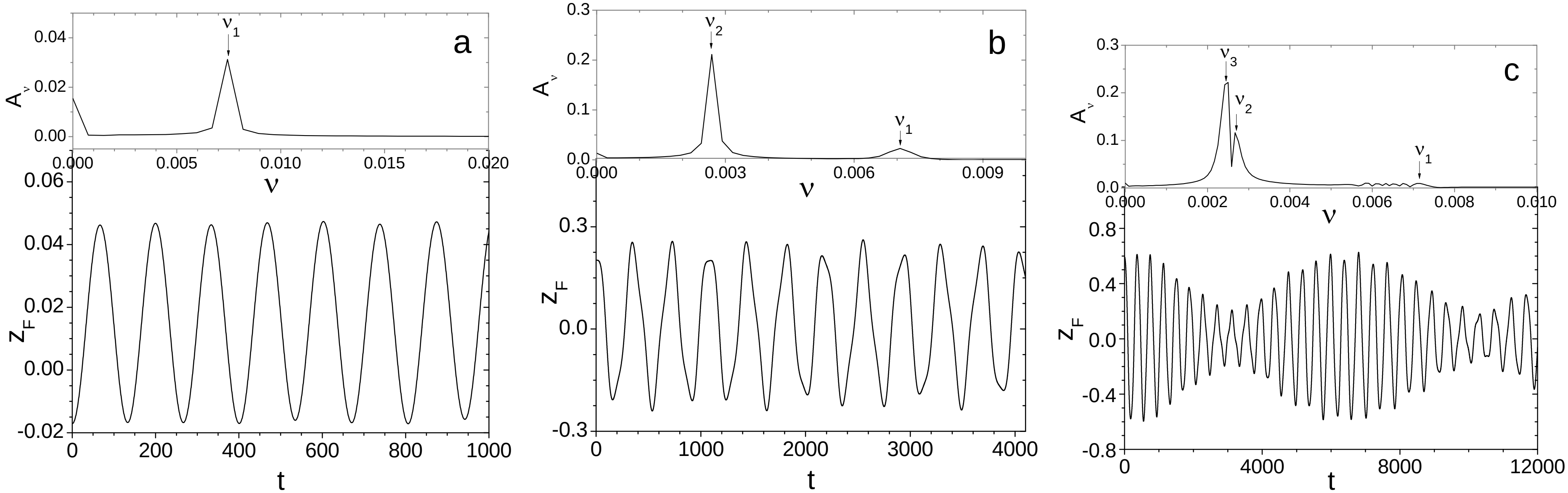
Panel a): BEC localized wavefunction $\Psi(x)$ (top curve) for $N_B = 470$ bosons and $N_F = 10$ fermions with attractive boson-boson and boson-fermion interactions $a_B/a_\perp = a_{BF}/a_\perp = -0.001$. Horizontal lines denote the first 30 fermionic energy levels. Panel b): lowest ten fermionic eigenfunctions $\chi_i(x)$ (from bottom to top) in the presence (continuous curves) and in the absence (dotted lines) of the BEC wavefunction depicted in panel a). [LS *et al.*, PRA **81**, 023614 (2010)]



Rabi linear frequency ν_α versus boson-fermion interaction a_{BF} for the first five fermionic doublets above the ground state with $N_F = 10$. The presence of BEC, with $N_B = 470$ and $a_B/a_\perp = -0.001$, reduces the quasi-degeneracy of doublets and this effect is stonger for the lowest doublet [LS *et al.*, PRA **81**, 023614 (2010)]



Fermionic density with $N_F = 10 + N_{ex}$, where 10 is the number of fermions in the ground state and N_{ex} is the number of excited fermions: $N_{ex} = 0$ (solid line), $N_{ex} = 1$ (dotted line), $N_{ex} = 2$ (dash dotted line), $N_{ex} = 3$ (dashed line). [LS *et al.*, PRA **81**, 023614 (2010)]



Dynamics of the fermionic density imbalance $Z_F(t)$ (bottom) and corresponding Fourier spectrum (top part of panels). Fermionic cloud of $N_F = 10 + N_{ex}$ fermions, where $N_{ex} = 1$ (a), $N_{ex} = 2$ (b) and $N_{ex} = 3$ (c) is the number of excited fermions. Bosonic cloud of $N_B = 470$ bosons and $a_B/a_\perp = a_{BF}/a_\perp = -0.001$. [LS *et al.*, PRA **81**, 023614 (2010)]

Conclusions

- We have investigated the tunneling dynamics of spin-polarized (ideal) fermions.

- Despite the fermions are not interacting the dynamics is quite complex: from periodic to strongly aperiodic by increasing the number N_F of fermions.

- The temperature T produces a spatial broadening of density profiles.

- We have also studied tunneling dynamics of fermions interacting with a localized BEC.

- The presence of a localized BEC modifies the Rabi frequencies ν_α of fermions.