Collective modes across the soliton-droplet crossover in binary Bose mixtures

Luca Salasnich

Dipartimento di Fisica e Astronomia "Galileo Galilei", Università di Padova
INO-CNR, Research Unit of Sesto Fiorentino, Consiglio Nazionale delle Ricerche

Laser Physics Workshop, Nottingham, July 18, 2018

Work done in collaboration with A. Cappellaro (Univ. Padova) and T. Macrì (Univ. Fed. Rio Grande do Norte)

Summary

- Introduction: self-bound Bose-Bose droplet
- Beyond-mean-field effective action
- Gaussian ansatz and soliton-droplet crossover
- Breathing modes
- Spin-dipole mode
- Conclusions
- Open problems

Introduction: self-bound Bose-Bose droplet

- Few years ago, Petrov[1] suggested theoretically the existence of self-bound quantum droplets in an attractive Bose-Bose mixture, where the collapse is suppressed by a beyond-mean-field term.
- Recent experiments with two internal states of ^{39}K atoms in a 3D configuration, performed both at Barcelona[2] and Florence[3], substantially confirm these theoretical predictions.
- In another experiment at Barcelona[4] self-bound states of the two-component BEC have been studied in a **tight optical waveguide** (quasi-1D confinement): a smooth crossover interpolating between bright soliton and droplet states has been observed.

In this talk I discuss our theoretical results[5] about this **soliton-droplet crossover**.

[1]D.S. Petrov, Phys. Rev. Lett. **115**, 155302 (2015).
[2]C.R. Cabrera *et al.*, Science **359**, 6373 (2018).
[3]G. Semeghini *et al.*. Phys. Rev. Lett. **120**, 235301 (2018)
[4]P. Cheiney *et al.*, Phys. Rev. Lett. **120** 135301 (2018).
[5]A. Cappellaro, T. Macrì, LS, Phys. Rev. A **97**, 053623 (2018).

Beyond-mean-field effective action (I)

We consider a Bose gas made of atoms in two different hyperfine states. Each component can be described by a complex field $\psi_j(\mathbf{r}, t)$ $(j = 1, 2)$, whose dynamics results from the following real-time low-energy effective action

$$S = \int dt\, d^3\mathbf{r} \Big[\sum_{j=1,2} \frac{i\hbar}{2} \left(\psi_j^* \partial_t \psi_j - \psi_j \partial_t \psi_j^* \right) - \mathcal{E}_{\mathrm{tot}}\left(\psi_1, \psi_2\right) \Big] \,. \qquad (1)$$

The total energy density $\mathcal{E}_{\mathrm{tot}}$ reads

$$\mathcal{E}_{\mathrm{tot}} = \sum_{j=1,2} \left[\frac{\hbar^2}{2m} |\nabla \psi_j|^2 + V_{\mathrm{ext}}(\mathbf{r}) |\psi_j|^2 + \frac{1}{2} g_{jj} |\psi_j|^4 \right] + g_{12} |\psi_1|^2 |\psi_2|^2 + \mathcal{E}_{\mathrm{BMF}}(\psi_1, \psi_2) \,, \qquad (2)$$

where $V_{\mathrm{ext}}(\mathbf{r})$ is an external confining potential, $g_{jk} = 4\pi\hbar^2 a_{jk}/m$ are the interaction strengths with a_{jk} being intra- and inter-species scattering lengths, and $n_j(\mathbf{r}, t) = |\psi_j(\mathbf{r}, t)|^2$ is the number density of the species j.

Beyond-mean-field effective action (II)

The beyond-mean-field term $\mathcal{E}_{\text{BMF}}$ arises from the zero-point energy of Bogoliubov elementary excitations[6], namely

$$\mathcal{E}_{\text{BMF}} = \frac{8}{15\pi^2}\left(\frac{m}{\hbar^2}\right)^{3/2}(g_{11}n_1)^{5/2} f\left(\frac{g_{12}^2}{g_{11}g_{22}}, \frac{g_{22}n_2}{g_{11}n_1}\right) \qquad (3)$$

with $f(x,y) = \sum_{\pm}[1 + y \pm \sqrt{(1-y)^2 + 4xy}]^{5/2}/(4\sqrt{2})$.
The calculation leading to the ground state properties can be simplified by assuming[7] the two components occupying the same spatial mode $\phi(\mathbf{r},t)$. The bosonic fields can then be written as

$$\psi_j(\mathbf{r},t) = \sqrt{N_j}\ \phi(\mathbf{r},t)\ . \qquad (4)$$

This assumption neglects the inter-component dynamics, resulting inadequate to probe spin-dipole oscillations.

---

[6] D.M. Larsen, Ann. Phys. 24, 89 (1963).

[7] D.S. Petrov, Phys. Rev. Lett. **115**, 155302 (2015); D.S. Petrov and G.E. Astrakharcik, Phys. Rev. Lett. **117**, 100401 (2016).

Beyond-mean-field effective action (III)

We work under the condition

$$\frac{N_1}{N_2} = \sqrt{\frac{a_{22}}{a_{11}}} \, , \tag{5}$$

which comes from the minimization of the mean-field energy density for the uniform system.[8] By defining[9]

$$\Delta a = a_{12} + \sqrt{a_{11} a_{22}} \quad \text{with} \quad a_{11} > 0, a_{22} > 0 \, , \tag{6}$$

the total energy density reads (with $N = N_1 + N_2$)

$$\begin{aligned} \frac{\mathcal{E}_{\text{tot}}}{N} &= \frac{\hbar^2}{2m} |\nabla \phi|^2 + V_{\text{ext}}(\mathbf{r}) |\phi|^2 + N \frac{4\pi\hbar^2}{m} \frac{\Delta a \sqrt{a_{22}/a_{11}}}{\left(1 + \sqrt{a_{22}/a_{11}}\right)^2} |\phi|^4 \\ &+ N^{3/2} \frac{256\sqrt{\pi}\hbar^2}{15m} \left(\frac{\sqrt{a_{11} a_{22}}}{1 + \sqrt{a_{22}/a_{11}}} \right)^{5/2} f\left(\frac{a_{12}^2}{a_{11} a_{22}}, \sqrt{\frac{a_{22}}{a_{11}}} \right) |\phi|^5 \, . \end{aligned} \tag{7}$$

[8] D.S. Petrov, Phys. Rev. Lett. **115**, 155302 (2015).

[9] Notice that the mean-field uniform system becomes unstable for $\Delta a < 0$.

Beyond-mean-field effective action (IV)

Inspired by the recent experiment of the Tarruell group at Barcelona[10] from now on, we consider a quasi-1D optical waveguide which gives a harmonic confinement **only** on a transverse plane

$$V_{\text{ext}}(\mathbf{r}) = \frac{1}{2} m\omega_\perp^2 (x^2 + y^2) \tag{8}$$

The presence of a harmonic potential defines a characteristic length scale, namely

$$a_\perp = \sqrt{\frac{\hbar}{m\omega_\perp}} \, . \tag{9}$$

We work with $a_{11} > 0$, $a_{22} > 0$ **but** $a_{12} < 0$. We investigate different values of

$$\Delta a = a_{12} + \sqrt{a_{11} a_{22}} \tag{10}$$

in the regime where $\Delta a < 0$, namely in the regime where $a_{12} < -\sqrt{a_{11} a_{22}}$.

[10] P. Cheiney, C.R. Cabrera, J. Sanz, B. Naylor, L. Tanzi, and L. Tarruell, Phys. Rev. Lett. **120** 135301 (2018).

Gaussian ansatz and soliton-droplet crossover (I)

The properties of the system can be analytically explored by taking a Gaussian variational ansatz

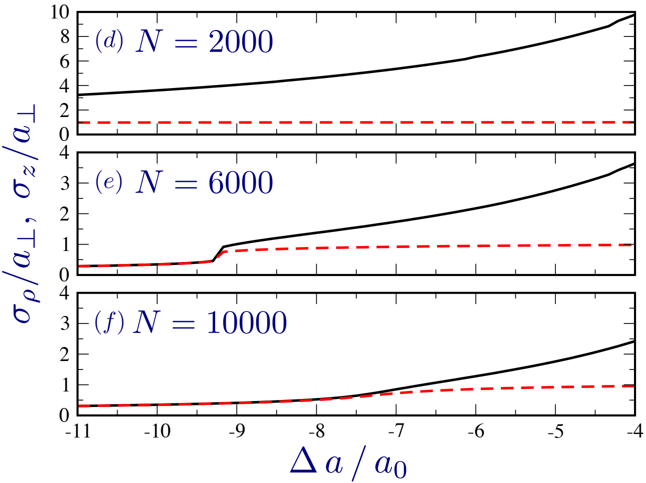
$$\phi(\mathbf{r}) = \frac{1}{\pi^{3/4}\sigma_\rho \sigma_z^{1/2}} \exp\left(- \frac{x^2+y^2}{2\sigma_\rho^2} - \frac{z^2}{2\sigma_z^2} \right) , \tag{11}$$

whose variational parameters are σ_ρ and σ_z.
By replacing Eq. (11) in the total energy density, the variational energy per particle is then given by

$$\frac{E}{N\hbar\omega_\perp} = \frac{1}{4}\left(\frac{2}{\sigma_\rho^2} + \frac{1}{\sigma_z^2}\right) + \frac{\sigma_\rho^2}{2} + \frac{2N\Delta a}{\sqrt{2\pi}\sigma_\rho^2\sigma_z} \frac{\sqrt{a_{22}/a_{11}}}{\left(1+\sqrt{a_{22}/a_{11}}\right)^2} + \frac{512\sqrt{2}}{75\sqrt{5}\pi^{7/4}} \frac{N^{3/2}}{(\sigma_\rho^2\sigma_z)^{3/2}} \left(\frac{\sqrt{a_{11}a_{22}}}{1+\sqrt{a_{22}/a_{11}}}\right)^{5/2} f\left(\frac{a_{12}^2}{a_{11}a_{22}}, \sqrt{\frac{a_{22}}{a_{11}}}\right) . \tag{12}$$

Here the lengths are in units of $a_\perp$.
Experimentally, by means of Feshbach resonance, below a critical value of the external magnetic field, the condition $\Delta a < 0$ is achieved.

Gaussian ansatz and soliton-droplet crossover (II)



Axial width σ_z (**black solid curve**) and transverse width σ_ρ (**red dashed curve**) as a function of Δa, for three values of the particle number N. We set $a_{11} = a_{22} > 0$ and consequently $\Delta a = a_{12} + |a_{11}|$. a_0 is the Bohr radius. **The self-bound spherical droplet is obtained when** $\sigma_\rho = \sigma_z$.

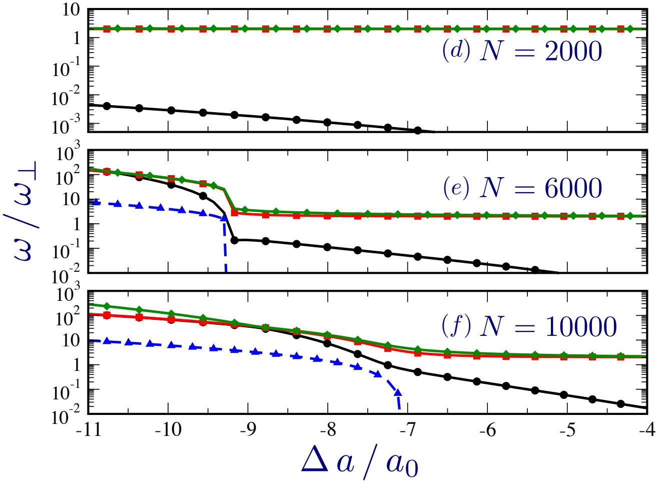
Breathing modes (I)

A deeper insight into the differences between solitonic and droplet states can be reached by examining collective excitations around the minima of the Gaussian variational energy.
Thus, we adopt a time-dependent Gaussian variational ansatz for the complex scalar field:

$$\phi(\mathbf{r},t) = \sqrt{\frac{1}{\pi^{3/2}\prod_{K=x,y,z}\sigma_K(t)}}\exp\left(\sum_{K=x,y,z}\left(-\frac{K^2}{2\sigma_K(t)^2}+i\beta(t)K^2\right)\right). \tag{13}$$

Inserting this ansatz into the beyond-mean-field effective action, after some manipulations we obtain a new effective action for the three time-dependent variational widths $\sigma_K(t)$.
From the corresponding linearized Euler-Lagrange equations we find three collective frequencies: two (quasi-degenerate) frequencies are related to a breathing oscillation in the $x-y$ plane while the third frequency is related to a beathing along the z axis.

Breathing modes (II)



Breathing frequencies (**red**, **green** and **black** curves) as a function of Δa, for three values of the particle number N. The **blue curve** is $-\mu/\hbar$, with μ the chemical potential for $\mu < 0$ (quantum droplet regime).

Spin-dipole mode (I)

We now consider the oscillatory motion occurring when there is a displacement $\bar{z}_1(t) - \bar{z}_2(t)$ along the z axis of the centers of mass $\bar{z}_j(t)$ of the two bosonic components.

To model this spin-dipole collective dynamics, we go beyond the assumption where the two components occupy the same spatial mode.

The corresponding Gaussian variational ansatz is given by

$$\psi_j(\mathbf{r},t) = \sqrt{\frac{N_j}{\pi^{3/2} \prod_{K=x,y,z} \sigma_K(t)}} \times \exp\left[\sum_{K=x,y,z} \left(-\frac{(K - \bar{z}_j(t)\delta_{K,z})^2}{2\sigma_{K,j}(t)^2} + i\alpha_j(t) z + i\beta_{K,j}(t) K^2 \right) \right],$$

where the fields $\psi_j(\mathbf{r},t)$ are normalized to N_j.

Spin-dipole mode (II)

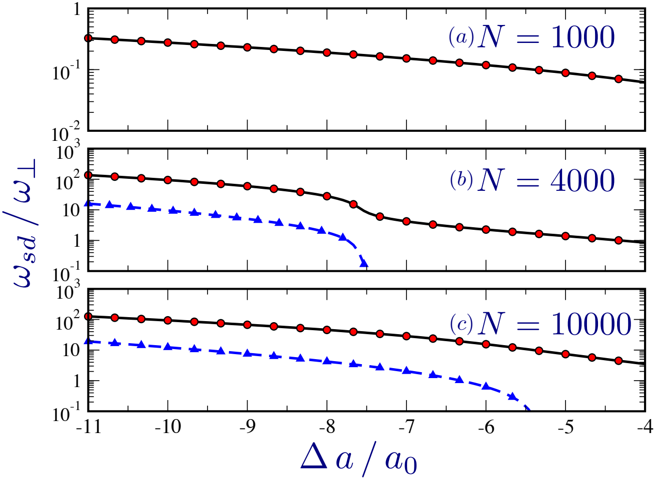
We insert the time-dependent variational ansatz for the two fields $\psi_j(\mathbf{r}, t)$ into the beyond-mean-field effective action, which becomes a functional of the variational parameters.
After several manipulations, we find[11] that the spin-dipole frequency ω_{sd} of the relative motion of the two bosonic clouds can be written as

$$\frac{\omega_{sd}^2}{\omega_\perp^2} = -\sqrt{\frac{8}{\pi}}\,\frac{N_1\left(a_{12}/a_\perp\right)}{\sigma_{\rho,0}^2\sigma_{z,0}^3} + \frac{2048}{25\pi^{1/4}}\,\frac{(a_{11}/a_\perp)^{5/2}N_1^{3/2}}{\sigma_{\rho,0}^2\sigma_{z,0}^{7/2}}\,, \qquad (14)$$

where $\sigma_{\rho,0}$ and $\sigma_{z,0}$ are the equilibrium values, and assuming $N_1 = N_2$.

[11] A. Cappellaro, T. Macri, LS, Phys. Rev. A **97**, 053623 (2018).

Spin-dipole mode (III)



Spin-dipole frequency (black curve with red dots) as a function of Δa, for three values of the particle number N. The **blue curve** is $-\mu/\hbar$ for $\mu < 0$.

Conclusions

- We have analyzed a two-component BEC with attractive interparticle interactions along the crossover from soliton to self-bound droplets in a quasi one dimensional waveguide.
- We have found a sharp difference of the collective modes in the two regimes:
  – in the soliton regime: two distinguishable collective frequencies;
  – deep into the spherical droplet regime: only one breathing frequency (triple degenerate).
- Also the spin-dipole mode can be excited, and we have found an analytical formula for it.

Open problems

- The experimental observability of collective modes in the quantum droplet (collective oscillations vs particle emission).
- The role of Rabi coupling on the properties of a two-component quantum droplet[12].
- The soliton-droplet crossover in a quasi-1D toroidal configuration.
- The formation of quantized vortices in a two-component quantum droplet.
- Two-component quantum droplets made of atoms with **different masses**.

[12] For recent theoretical results see: A. Cappellaro, T. Macri, G.F. Bertacco, and LS, Sci Rep. **7**, 13358 (2017).

Acknowledgements

Thank you for attention!

Main sponsor: Italian Ministry of Education, University and Research (FFABR Grant).