Bright solitons of attractive Bose-Einstein condensates confined in a quasi-1D optical lattice

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Altea, September 20, 2013

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Summary

- BEC in a quasi-1D optical lattice
- Axial discretization of the 3D GPE
- Transverse dimensional reduction of the 3D DGPE
- 1D DNPSE
- Numerical results
- Collapse of the discrete bright soliton
- Conclusions

BEC in a quasi-1D optical lattice (I)

We consider a dilute Bose-Einstein condensate (BEC) confined in the z direction by a **generic axial potential** $V(z)$ and in the plane (x, y) by the **transverse harmonic potential**

$$U(x, y) = \frac{m}{2}\omega_\perp^2 \left(x^2 + y^2\right) \ . \tag{1}$$

The characteristic harmonic length is given by

$$a_\perp = \sqrt{\frac{\hbar}{m\omega_\perp}} \ , \tag{2}$$

and, for simplicity, we choose $a_\perp$ and $\omega_\perp^{-1}$, as length and time units, and $\hbar\omega_\perp$ as energy unit.

BEC in a quasi-1D optical lattice (II)

We assume that the system is well described by the 3D Gross-Pitaevskii equation (GPE), and in scaled units it reads

$$i\frac{\partial}{\partial t}\psi(\mathbf{r},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2}\left(x^2+y^2\right) + V(z) + 2\pi g|\psi(\mathbf{r},t)|^2\right]\psi(\mathbf{r},t)\ , \tag{3}$$

where $\psi(\mathbf{r},t)$ is the macroscopic wave function of the condensate normalized to the total number N of atoms and $g = 2a_s/a_\perp$ with a_s the s-wave scattering length of the inter-atomic potential.
In addition, we suppose that **the axial potential is the combination of periodic and harmonic potentials**, i.e.

$$V(z) = V_0\cos(2kz) + \frac{1}{2}\lambda^2 z^2\ . \tag{4}$$

This potential models the **quasi-1D optical lattice** produced in experiments with Bose-Einstein condensates by using counter-propagating laser beams.[1] Here $\lambda = \omega_z/\omega_\perp \ll 1$ models a weak axial harmonic confinement.

[1] O. Morsch and M. Oberthaler, Rev. Mod. Phys. **78**, 179 (2006).

Axial discretization of the 3D GPE (I)

We now perform a discretization of the 3D GPE along the z axis due to the presence on the periodic potential. In particular we set

$$\psi(\mathbf{r}, t) = \sum_n \phi_n(x, y, t)\ W_n(z) \tag{5}$$

where $W_n(z)$ is the **Wannier function** maximally localized at the n-th minimum of the axial periodic potential. This tight-binding ansatz is reliable in the case of a deep optical lattice.[2]

[2] A. Smerzi and A. Trombettoni, Phys. Rev. A **68**, 023613 (2003).

Axial discretization of the 3D GPE (II)

We insert this ansatz into Eq. (3), multiply the resulting equation by $W_n^*(z)$ and integrate over z variable. In this way we get

$$i\frac{\partial}{\partial t}\phi_n = \left[-\frac{1}{2}\nabla_\perp^2 + \frac{1}{2}\left(x^2+y^2\right) + \epsilon_n\right]\phi_n - J\left(\phi_{n+1}+\phi_{n-1}\right) + 2\pi U\left|\phi_n\right|^2\phi_n, \tag{6}$$

where the parameters ϵ, J and U are given by

$$\epsilon_n = \int W_n^*(z)\left[-\frac{1}{2}\frac{\partial^2}{\partial z^2} + V(z)\right]W_n(z)\ dz\ , \tag{7}$$

$$J = -\int W_{n+1}^*(z)\left[-\frac{1}{2}\frac{\partial^2}{\partial z^2} + V(z)\right]W_n(z)\ dz\ , \tag{8}$$

$$U = g\int \left|W_n(z)\right|^4\ dz\ . \tag{9}$$

The parameters J and U are practically independent on the site index n and in the tight-binding regime $J > 0$.

Transverse dimensional reduction of the 3D DGPE

To further simplify the problem we set[3]

$$\phi_n(x,y) = \frac{1}{\pi^{1/2}\sigma_n(t)} \exp\left[-\left(\frac{x^2+y^2}{2\sigma_n(t)^2}\right)\right] f_n(t) \,, \tag{10}$$

where $\sigma_n(t)$ and $f_n(t)$, which account for **discrete transverse width** and **discrete axial wave function**, are the effective generalized coordinates to be determined variationally.

We insert this ansatz into the Lagrangian density associated to Eq. (6) and integrate over x and y variables. In this way we obtain an effective Lagrangian for the fields $f_n(t)$ and $\sigma_n(t)$.

[3] A. Maluckov, L. Hadzievski, B.A. Malomed, LS, Phys. Rev. A **78**, 013616 (2008); G. Gligoric, A. Maluckov, LS, B. A. Malomed, L. Hadzievski, Chaos **19**, 043105 (2009).

1D DNPSE (I)

The Euler-Lagrange equation of the effective Lagrangian with respect to f_n^* is

$$i\frac{\partial}{\partial t}f_n = \left[\frac{1}{2}\left(\frac{1}{\sigma_n^2}+\sigma_n^2\right)+\epsilon_n\right] f_n - J\left(f_{n+1}+f_{n-1}\right) + \frac{U}{\sigma_n^2}|f_n|^2 f_n \ . \qquad (11)$$

while the Euler-Lagrange equation with respect to σ_n gives

$$\sigma_n^4 = 1 + U|f_n|^2 \ . \qquad (12)$$

Inserting Eq. (12) into Eq. (11) we finally get

$$i\frac{\partial}{\partial t}f_n = \epsilon_n \, f_n - J\left(f_{n+1}+f_{n-1}\right) + \frac{1+(3/2)U|f_n|^2}{\sqrt{1+U|f_n|^2}} f_n \ , \qquad (13)$$

that is the 1D discrete nonpolynomial Schrödinger equation (DNPSE), describing the BEC under a transverse anisotropic harmonic confinement and an axial optical lattice.

1D DNPSE (II)

The 1D NPSE reduces to the familiar 1D DGPE (1D cubic DNLSE)

$$i\frac{\partial}{\partial t}f_n = \epsilon_n\, f_n - J\,\left(f_{n+1} + f_{n-1}\right) + U|f_n|^2 f_n \tag{14}$$

in the weak-coupling limit $|U||f_n|^2 \ll 1$, where U can be both positive and negative. On the contrary, it becomes a 1D quadratic DNLSE

$$i\frac{\partial}{\partial t}f_n = \epsilon_n\, f_n - J\,\left(f_{n+1} + f_{n-1}\right) + (3/2)\sqrt{U}|f_n|f_n \tag{15}$$

in the strong-coupling limit $U|f_n|^2 \gg 1$, where $U > 0$.

Numerical results (I)

We have solved numerically both 1D DNPSE and 1D DGPE by using a Crank-Nicolson predictor-corrector algorithm with imaginary time to get the ground-state of the system.

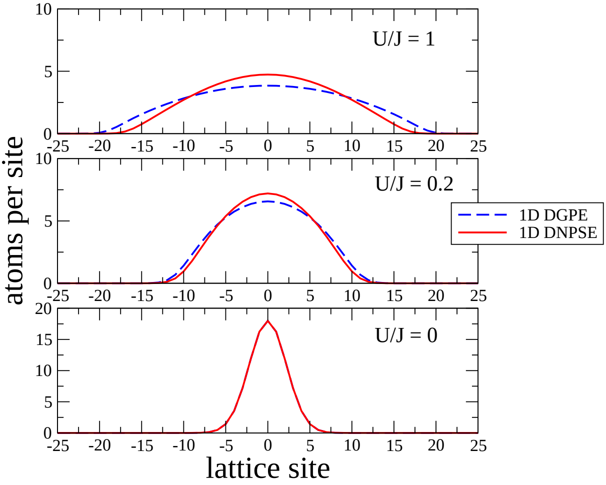
In the next two slides we report our results obtained with $N = 100$ atoms in a quasi-1D optical lattice with weak axial harmonic confinement: $\lambda = \omega_z/\omega_\perp = 0.1$.

The plots are shown for different values of the **repulsive** on-site interaction strength U: $U > 0$.

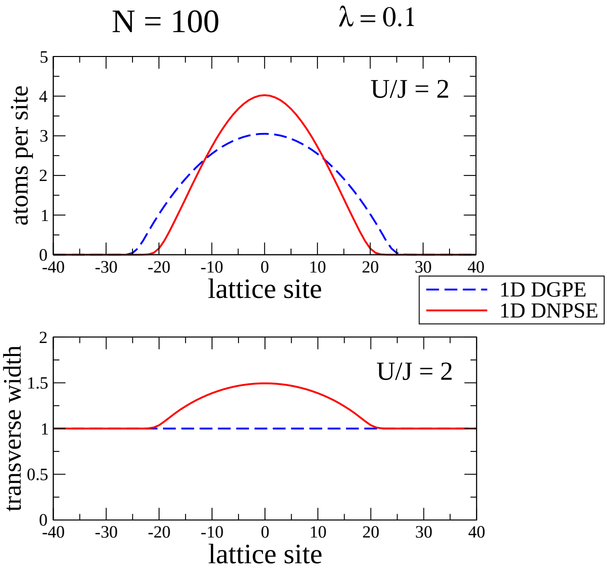
In the experiments U can be tuned by using the technique of Feshbach resonances.

Numerical results (II)

$N = 100$ $\lambda = 0.1$



Numerical results (III)

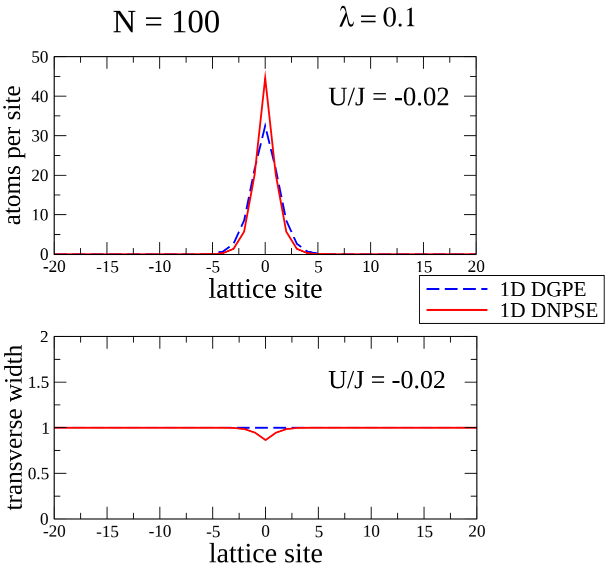


Numerical results (IV)

Now we show the results obtained again with $N = 100$ atoms in a quasi-1D optical lattice but with an **attractive** on-site interaction strength U: $U < 0$.

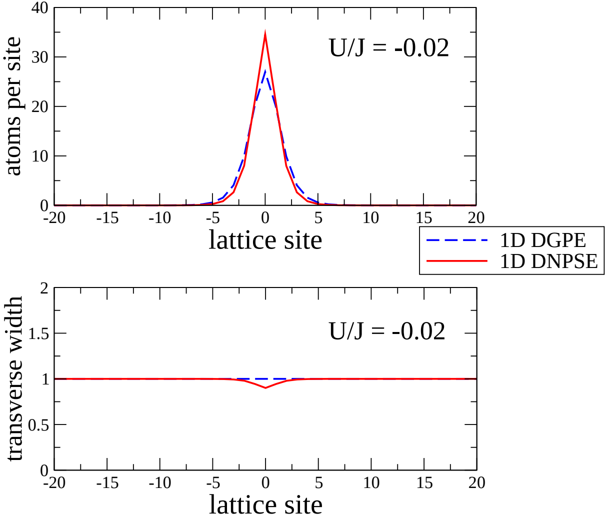
In the attractive case the ground-state is self-localized and it exists also in the absence ($\lambda = 0$) of the axial harmonic potential: discrete bright soliton.

Numerical results (V)



Numerical results (VI)

$N = 100$ $\lambda = 0$ (no axial trap)



Collapse of the discrete bright soliton (I)

By increasing the attractive on-site interaction U ($U < 0$) the 1D DGPE shows that eventually all the atoms accumulate into the same site.

Actually, the 1D DNPSE shows something different: before all the atoms populate the same site there is the **collapse of the condensate**: 1D DNPSE does not admit anymore a finite ground-state solution.

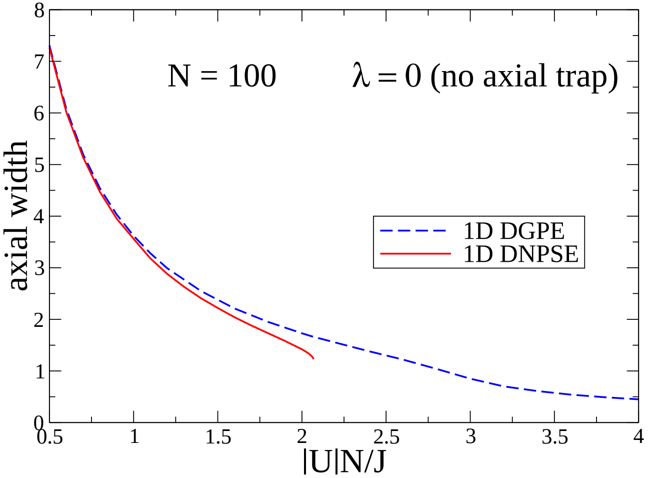
Numerically we find that the collapse occurs when $U < 0$ and

$$\frac{|U|N}{J} \gtrsim 2.1 \tag{16}$$

which is consistent with analytical result[4] $|U|N/J > 8/3$ of the continuum limit.

[4] LS, A. Parola, L. Reatto, Phys. Rev. A **65**, 043614 (2002).

Collapse of the discrete bright soliton (II)



Conclusions

- From the 3D GPE of bosons in a quasi-1D optical lattive we have derived an effective 1D DNPSE.
- The DNPSE reduces to the 1D DGPE in the weak-coupling limit.
- The DNPSE gives quite different results with respect to the 1D DGPE in the (repulsive) strong-coupling limit.
- In the case of attractive on-site interaction there is a self-localized solution: the discrete bright soliton.
- The DNPSE predicts the collapse of the discrete bright soliton above a critical (attractive) on-site interaction.
- Our results are reliable in the superfluid regime $|U|N/J \ll N^2$ where the 3D GPE is meaningful.

Acknowledgments

THANK YOU FOR YOUR ATTENTION!

We acknowledge research grants from:
– Università di Padova: Progetto di Ricerca di Ateneo 2012-2013.
– Fondazione CARIPARO: Progetto di Eccellenza 2012-2013.
– MIUR: Progetto PRIN 2012-2014.