# QUANTIZED VORTICES

LET US CONSIDER THE STATIONARY GPE :

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + U(\rho,z) + g|\psi|^2\right]\psi = \bar{\mu}\,\psi$$

A QUANTIZED VORTEX CAN BE DESCRIBED SETTING

$$\psi(\rho,z,\theta) = \varphi(\rho,z)\,e^{iK\theta} \qquad K\in\mathbb{Z} \text{ AND } \varphi(\rho,z)\in\mathbb{R}. \quad \theta\in[0,2\pi]$$

IN THIS WAY THE GPE BECOMES

$$\left[-\frac{\hbar^2}{2m}\nabla_\perp^2 + \frac{\hbar^2K^2}{2m\rho^2} + U(\rho,z) + g\,\varphi^2\right]\varphi = \bar{\mu}\,\varphi\,.$$

NOTE THAT THE INTEGER NUMBER K IS <u>THE QUANTUM OF CIRCULATION</u> AND THE ANGULAR MOMENTUM ALONG $z$ IS $N\hbar K$.

NOTE THAT $\rho, z, \theta$ ARE CYLINDRIC COORDINATES.

IF THE EXTERNAL POTENTIAL IS ZERO $U(\rho,z)=0$

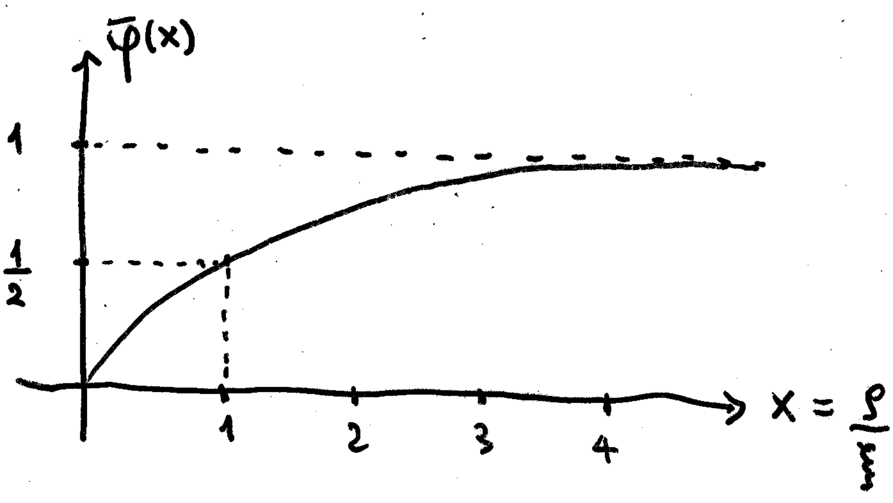
IMPOSING THAT $\varphi^2 \to n$ FOR $\rho\to\infty$

ONE FINDS $\bar{\mu} = gn$.

SETTING $x = \frac{\rho}{\xi}$ WHERE $\xi = \sqrt{\frac{\hbar^2}{2m\bar{\mu}}}$ HEALING LENGTH

THE EQUATION FOR $\bar{\varphi}(x) = \varphi(\rho/\xi)\frac{1}{\sqrt{n}}$ READS

$$\frac{1}{x}\frac{d}{dx}\left(x\frac{d}{dx}\bar{\varphi}\right) - \frac{\bar{\varphi}}{x^2} + \bar{\varphi} - \bar{\varphi}^3 = 0 \qquad \boxed{K=1}$$




$\bar{\varphi}(x) \sim x$ FOR $x\to 0$

$\bar{\varphi}(x) \sim 1 - \frac{1}{2x^2}$ FOR $x\to+\infty$

# QUANTIZED VORTEX OBTAINED WITH GPE

——— $K = 0$

——— $K = 1$

- - - - - $K = 1$ (NON INTERACTING BEC)

$$\oint dS = 2\pi K$$

SINGLE-VALUED WAVE FUNCTION

NOTE THAT

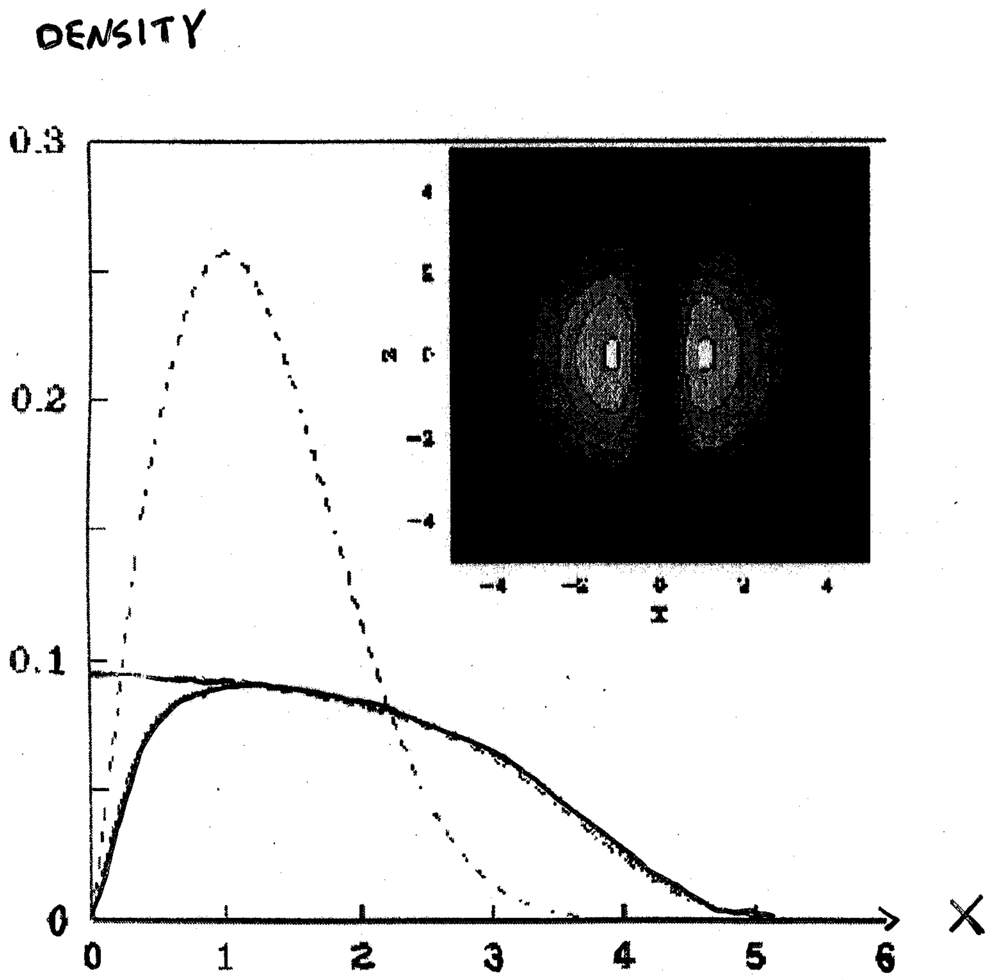
$$\oint \vec{v} \cdot d\vec{r} = \frac{K h}{m}$$

ONSAGER - FEYNMAN QUANTIZATION CONDITION

WHERE $\vec{v} = \vec{\nabla} S \left(\frac{\hbar}{m}\right)$ AND

$$\psi = |\psi| e^{iS}$$

$$S = K\theta$$




IF $\vec{v} = \vec{\nabla} \alpha$

THEN

$$\vec{\nabla} \wedge \vec{v} = \vec{0}$$

AND THE FLUID IS CALLED <u>IRROTATIONAL</u>

NOW WE CONSIDER A NON ZERO EXTERNAL POTENTIAL

$U(\rho, z) \neq 0$.

IF THE NUMBER $N$ OF BOSONS IS LARGE WE CAN USE THE THOMAS-FERMI APPROXIMATION OF THE GPE EQUATION WITH $K \neq 0$

$$\left[ -\frac{\hbar^2}{2m} \nabla_\rho^2 + \frac{\hbar^2 K^2}{2m\rho^2} + U(\rho, z) + g\,\varphi^2 \right] \varphi = \bar{\mu}\,\varphi$$

FINDING

$$n(\rho, z) = \varphi(\rho,z)^2 = \frac{1}{g}\left( \bar{\mu} - \frac{\hbar^2 K^2}{2m\rho^2} - U(\rho,z) \right).$$

- USUALLY THE VORTEX STATE IS AN EXCITED STATE OF THE SYSTEM, BUT IT CAN BECOME THE GROUND STATE IF THE SYSTEM IS SET INTO ROTATION. LET $\Omega$ BE THE FREQUENCY OF ROTATION ALONG THE $z$ AXIS. IN THE ROTATING FRAME THE ENERGY FUNCTIONAL TO BE MINIMIZED IS

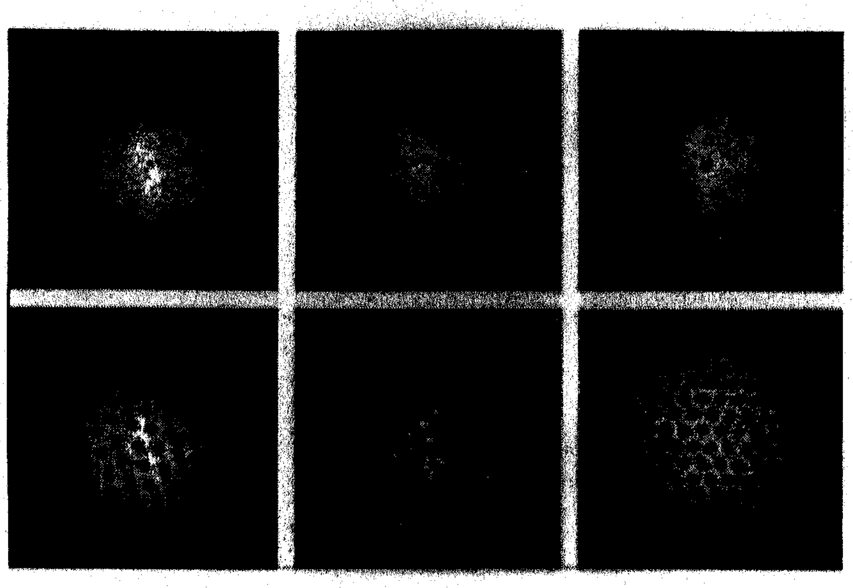
$$E_\Omega[\psi] = \int d^3r\, \psi^* \left[ -\frac{\hbar^2}{2m}\nabla^2 + V + \frac{1}{2} g |\psi|^2 \right] \psi - \Omega \int d^3r\, \psi^* \hat{L}_z \psi$$

WHERE $\hat{L}_z = -i\hbar \frac{\partial}{\partial \theta}$.

IT IS POSSIBLE TO PROVE THAT:

- FOR $0 \leq \Omega \leq \Omega_c \implies$ NO VORTICES
- FOR $\Omega_c < \Omega \leq \Omega_{cc} \implies$ ONE VORTEX WITH $K=1$
- FOR $\Omega_{cc} < \Omega \leq \Omega_{ccc} \implies$ TWO VORTICES WITH $K=1$

MULTIPLE QUANTIZED VORTICES HAVE BEEN OBSERVED IN VARIOUS EXPERIMENTS WITH BECs.



FROM: V. SCHWEIKHARD ET AL., PRL 93, 210403 (2004)

FORMATION OF MANY VORTICES (VORTEX LATTICE) BY INCREASING THE ANGULAR VELOCITY $\Omega$