EXPERIMENTAL QUANTUM GASES

• Introduction

Bose–Einstein condensation with ultracold atoms first achieved in 1995.

1924 : paper of Satyendra Nath Bose on quantum statistics of photons
Bose sent it to A. Einstein for comments and to get it traslated in German (he traslated A.E. papers in English) for publication in Zeitschrift für Physik

1925 : A.E. realized Bose analysis could be extended to particles with masses
striking prediction : below T_c, finite fraction of particles in same quantum state

1908 : Kamerlingh Onnes liquified ^{4}He

1938 : London suggested superfluidity of ^{4}He connected to BEC
SF He only candidate for BEC for a long time

Now we know that, in liquid He, BEC fraction ~ 10% (n-scattering) due to strong interactions

1959 : idea to condense spin-polarized H atoms
H atoms with aligned spin cannot form a molecule
→ use H atoms in B field

late '70s : research on cold H cooled by contact to cold surface
limited density → development of magnetic traps and evaporative cooling

1998 : eventually BEC in H

But in the meantime

1995 : BEC in alkali atoms (Rb, Na, Li)
Thanks to laser cooling, developed in '80s from original proposals of 1975

Alkali atoms much heavier than H but their optical transitions from ground state much more convenient, wavelength in near-infrared region accessible with lasers.

Lyman-α line in H, $1s \rightarrow 2p$, $\lambda = 121$ nm.

Early '90s : laser cooling $\longrightarrow$ temperatures $\sim 1\,\mu K$
but roadblock
lower temperatures NOT possible because of single-phot. recoil energy $\sim 100\,nK$
higher densities NOT possible because effective repulsion between atoms due to absorption/emission of photons

1995 : BEC in Rb, combination of laser cooling + evaporative cooling

Quantum degeneracy, BEC

$$n \lambda_{dB}^3 \sim 1 \qquad \lambda_{dB} = \frac{\hbar}{\sqrt{2\pi m k_B T}}$$

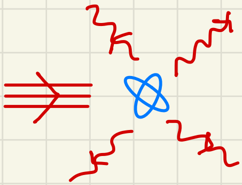
particles in phase-space volume $\hbar^3 \sim 1$

Gas at room temperature $n \lambda_{dB}^3 \sim 10^{-14}$

• Laser cooling

Original idea 1975 by Hansch & Schawlow and Wineland & Dehmelt independently (all Nobel laureates, none for laser cooling)

Radiation pressure:



$$\Delta \vec{p} = \hbar \vec{k}_L - \hbar \vec{k}_{s.e.}$$

$$\vec{F} = \frac{1}{\Delta t} \langle \Delta \vec{p} \rangle = \hbar \vec{k}_L \cdot R_{abs}$$

absorption rate

Spontaneous emission, random direction $\langle \hbar \vec{k}_{s.e.} \rangle = 0$

Recoil velocity (Rb) $\simeq 6$ mm/s

Absorption rate, at large $I \simeq \frac{1}{2\tau} \simeq 2 \cdot 10^7 \ s^{-1}$

$\rightarrow$ Acceleration $\simeq 6 \cdot 10^{-3} \cdot 2 \cdot 10^7 \ \frac{m}{s^2} = 1.2 \cdot 10^5 \ \frac{m}{s^2} = 1.2 \cdot 10^4 g$

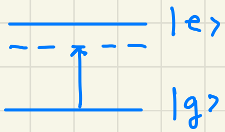
Thermal velocity $\sqrt{\frac{k_B T}{m}} \simeq 240$ m/s

$\rightarrow$ deceleration time $\frac{240}{1.2} \cdot 10^{-5} = 2$ ms

deceleration distance $= \sqrt{\frac{v^2}{2a}} \simeq 0.5$ m

Stopping distance $\gg$ laser beam size.
In experiments only low-velocity tail of Maxwell-Boltzmann can be cooled.

Basic principle (two level atom)



$$\Delta = \hbar\omega_0 - \hbar\omega_L, \quad \vec{E}_L(\vec{r},t) = \hat{\epsilon}\, E_0(\vec{r}) \cos(\vec{k}\vec{r} - \omega_L t)$$

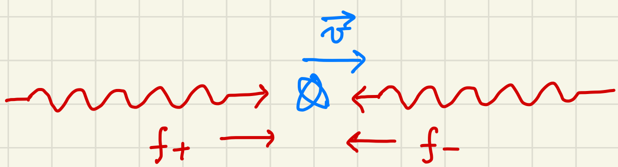
Photons scattering rate $= \Gamma \rho_{ee}$

$$\rho_{ee} = \frac{1}{2} \frac{\Omega^2/2}{\Omega^2/2 + (\Gamma/2)^2 + \Delta^2}, \quad \Omega = -\frac{\langle e|d|g\rangle E_0}{\hbar}$$

Ω Rabi frequency, $\dfrac{(\Omega^2/2)}{(\Gamma^2/4)} = \dfrac{I}{I_s}$ $\quad I_s$ Saturation intensity

At $\Delta = 0$, $I \gg I_s$: $\rho_{ee} = \dfrac{1}{2} \dfrac{I/I_s}{1 + I/I_s + 4\Delta^2/\Gamma^2} \rightarrow \dfrac{1}{2}$ max value

<u>Doppler cooling</u>



Atom reference frame $\quad \omega_L' = \omega_L - \vec{k}_L \cdot \vec{v}$

$$\Delta' = \omega_L' - \omega_0 = \Delta - \vec{k}_L \cdot \vec{v}$$

Individual forces (one beam alone)

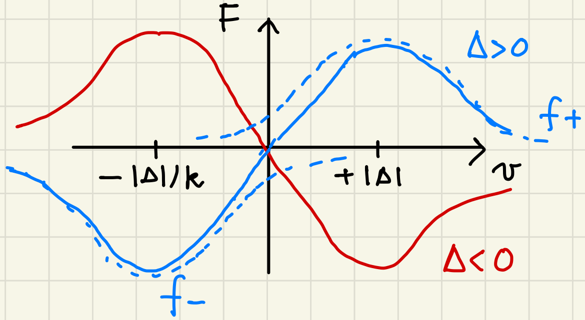
$$f_\pm = \pm \hbar k_L \frac{\Gamma}{2} \frac{I/I_s}{1 + I/I_s + 4(\Delta \mp k_L v)^2/\Gamma^2}$$

At low intensities $I/I_s \ll 1$, sum individual forces

$$F = f_+ + f_- = \hbar k_L \frac{\Gamma}{2}\left[\frac{I/I_s}{1+I/I_s+4(\Delta-k_L v)^2/\Gamma^2} - \right.$$
$$\left. - \frac{I/I_s}{1+I/I_s+4(\Delta+k_L v)^2/\Gamma^2}\right]$$

$$\underset{O(I/I_s)}{\simeq} \hbar k_L \frac{\Gamma}{2}\frac{I}{I_s}\frac{16\,\Delta k v/\Gamma^2}{1+16\left((\Delta^2-k_L^2v^2)^2/\Gamma^2\right)^2+8(\Delta^2+k_L^2v^2)/\Gamma^2}$$

Net force is viscous for $\Delta<0$



The velocities around $v=0$ are damped for $\Delta<0$

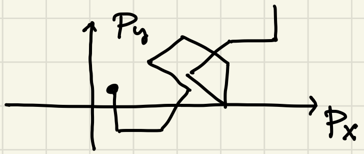
$$F(v) = -\alpha(\Delta)\, v \qquad (\text{for small velocities})$$

$\longrightarrow$ Doppler cooling

Lower limit on temperature:

spontaneous emission is random

random walk / diffusion in momentum space



$$(\Delta p)^2 = D\cdot t \simeq$$
$$\simeq (\hbar k_L)^2 R\cdot t$$

The equilibrium temperature is

$$T \propto \frac{D}{\alpha(\Delta)} \qquad \text{fluctuation / dissipation}$$

$$\min(T) \equiv T_D = \frac{1}{k_B}\frac{\hbar\Gamma}{2} \qquad \text{Doppler temperature}$$

For Rb, $T_D \simeq 140\ \mu K$

"Optical molasses", actual temperatures can be $\ll T_D$ in multi-level atoms

Ultimate limit, recoil temperature $T_R = \frac{(\hbar k_L)^2}{2m}\frac{1}{k_B}$

- Magneto-optical traps

Radiation pressure forces alone cannot create a trap with static laser beams.

Multi-level atoms:

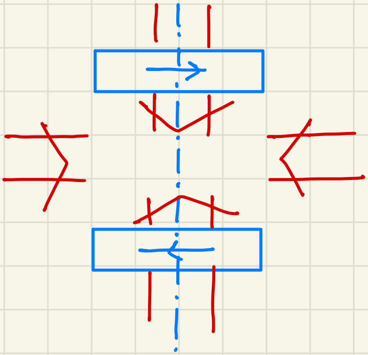
inhomogeneous magnetic fields + beams with different polarization

$$\longrightarrow \quad F = -\alpha v - \beta x$$

additional elastic term

Proposed by J. Dalibard (1986), later realized by Steven Chu (Nobel laureate 1997).

Real MOT (in a vacuum chamber)

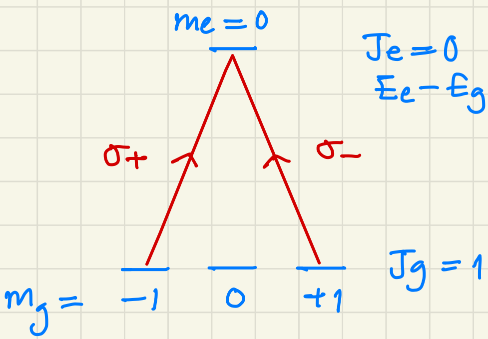


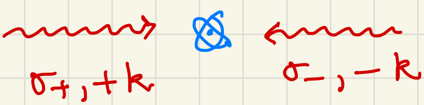
3 pairs of counterpropagating beams in a magnetic quadrupolar field

$$\vec{B}(\vec{r}) = b\left(-x\hat{x} - y\hat{y} + 2z\hat{z}\right)$$

The MOT cannot be understood with two-level atoms, it requires taking into account polarization of light and the associated selection rules of transitions.

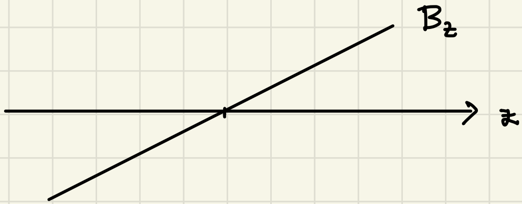
As a minimal model, consider a $J=1\to 0$ transition and a 1D configuration with only two laser laser beams with opposite circular polarizations and directions

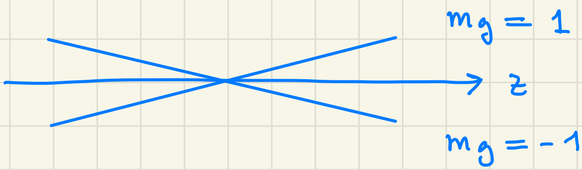




Add an inhomogeneous magnetic field : $B_z = b\cdot z$ and consider the associated Zeeman effect

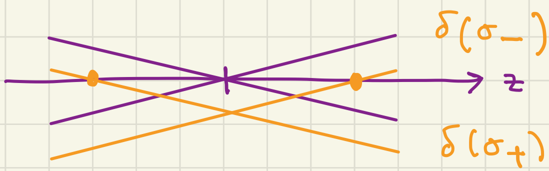
$$\Delta E\,(m_g, m_e = 0) = 0 \;, \quad \Delta E\,(m_g = \pm 1) = \mu_B\, g_J \cdot (\pm 1)\, b\cdot z$$





As a consequence the frequency of the transition is also z-dependent, and so are the detunings:

$$\delta(\sigma_+) = \omega_L - \omega_0 + \Delta E(m_g = -1)/\hbar$$
$$= \delta_L - \mu_0 g_J b z/\hbar = \delta_L - \beta z \qquad \beta \equiv \frac{\mu_0 g_J b}{\hbar}$$
$$\delta(\sigma_-) = \delta_L + \mu_0 g_J b z/\hbar = \delta_L + \beta z$$



Therefore the force f_+ due to the σ_+ beam becomes

$$f_+ = \hbar k_L \frac{\Gamma}{2}\left[\frac{I/I_s}{1 + I/I_s + 4(\Delta - \beta z - k_L v)^2/\Gamma^2}\right]$$

Similarly the force f_- due to the σ_- beam becomes

$$f_- = -\hbar k_L \frac{\Gamma}{2}\left[\frac{I/I_s}{1 + I/I_s + 4(\Delta + \beta z + k_L v)^2/\Gamma^2}\right]$$

The sum force, at lowest order in I/I_s, $k_L v/\Gamma$, $\beta z/\Gamma$ is:

$$F = f_+ + f_- \simeq \hbar k_L \frac{\Gamma}{2} \frac{I}{I_s} \times \frac{16\Delta\,(k_L v + \beta z)/\Gamma^2}{(1 + 4\Delta^2/\Gamma^2)^2}$$

Thus we added an harmonic force to the viscous force.

Magnetic traps

Laser cooling is insufficient (in most cases) to reach quantum degeneracy.

A second stage of cooling is usually performed by evaporation, for this the gas needs to be trapped for a few to a few hundred seconds.

Two type of traps are generally used: "dipole" trap (based on the shift of energy levels from a detuned laser beam, seen earlier) or "magnetic" trap.

- **Zeeman effect**

Magnetic traps use the Zeeman effect, combined with inhomogeneous magnetic fields.

In alkali atoms, ground electronic level has a hyperfine structure mainly due to the coupling of the valence electron and nucleus magnetic moments:

$$\mathcal{H}_{Hf} = A\vec{J}\cdot\vec{I}$$

$\vec{J}, \vec{I}$ electron / nuclear angular momentum

In ground state: $\underbrace{S = 1/2}_{\text{spin}}, \ \underbrace{L = 0}_{\text{orbital}} \rightarrow J = 1/2$

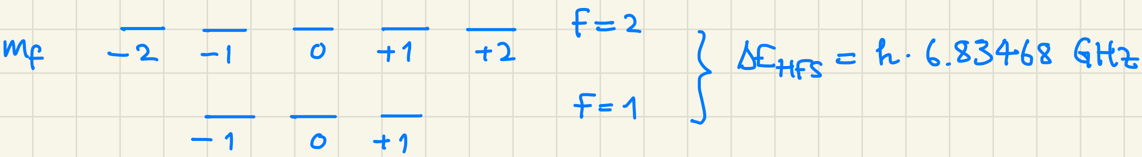
orbital, spin: angular momentum of electron

Defining $\vec{F} = \vec{I} + \vec{J}$

$$\mathcal{H}_{HF} = A \cdot \frac{1}{2} \left[\vec{F}^2 - \vec{J}^2 - \vec{I}^2 \right] =$$

$$= \frac{1}{2} A \left[F(F+1) - I(I+1) - \frac{3}{4} \right]$$

The hyperfine hamiltonian is diagonal in basis $|F, m_F\rangle$, for example ^{87}Rb $(I = 3/2)$



The remaining degeneracy is lifted by a magnetic field, through Zeeman effect

$$\mathcal{H}_Z = -\vec{\mu} \cdot \vec{B} = B_z \, \underbrace{g_J}_{\text{gyromagnetic factor}} \cdot J_z \, \underbrace{\mu_B}_{\text{Bohr magneton } \frac{e\hbar}{2mc}}$$

Nuclear magnetic moment is smaller by a factor $\sim \frac{m_e}{m_N}$ and therefore neglected.

Zeeman hamiltonian is treated at 1st order perturbation theory when $\mu_B \cdot B \ll \Delta E_{HF}$

$$\Delta E_Z = \langle F, m_F | \mu_B B g_J J_z | F, m_F \rangle = \mu_B B g_J \langle J_z \rangle$$

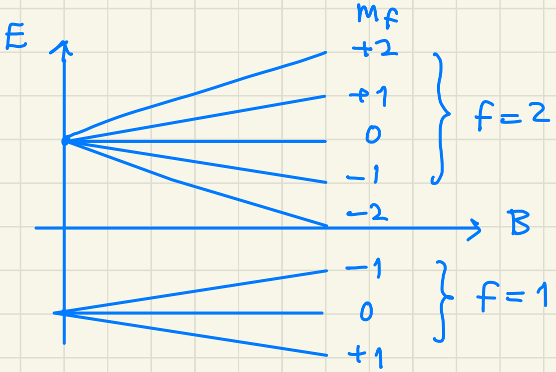
By Wigner-Eckard theorem $\langle J_z \rangle$ is proportional to $\langle F_z \rangle$

$$\Delta E_z = \mu_B B \, g_F \langle F_z \rangle$$

$$\longrightarrow \quad g_F = g_J \frac{1}{2} \left[\frac{F(F+1) + J(J+1) - I(I+1)}{F(F+1)} \right]$$

Since $J = S$, electron angular momentum is spin, $g_J = 2$:

$$g_{F=2} = \frac{1}{2}, \quad g_{F=1} = -\frac{1}{2}$$



We have taken quantization axis along the magnetic field.

(*) For a inhomogeneous B field, the direction of $\vec{B}$ can depend on position $\vec{r}$: the eigenstate $|F, m_F\rangle$ is not the same state at different locations $\vec{r}_1, \vec{r}_2$

$$|F, m_F\rangle_{\vec{r}_1} \neq |F, m_F\rangle_{\vec{r}_2}$$

Under "adiabatic approximation", the state of an atom is $|F, m_F\rangle_{\vec{r}_1}$ in $\vec{r}_1$ and $|F, m_F\rangle_{\vec{r}_2}$ in $\vec{r}_2$.

Adiabatic approximation requires that the direction of magnetic field (= quantization axis) changes slowlly

$$\left| \frac{d}{dt} \hat{b} \right| \ll \text{energy separations} / \hbar$$
$$\sim \frac{\mu_B B}{\hbar}$$

Therefore within adiabatic approximation, a magnetic field creates a potential energy

$$U_{(F,m_F)}(\vec{r}) = \mu_B \, g_F \, m_F \, |\vec{B}|$$

For states $|F, m_F\rangle$ such that $g_F m_F > 0$, a point of minimum $|\vec{B}|$ is also a potential minimum, i.e. a trap. These states are said "low-field seekers".

Viceversa, states with $g_F m_F < 0$ are trapped around maxima of $|\vec{B}|$, these are "high-field seekers".

<u>Wing's theorem</u>: in absence of sources, a point of maximum $|\vec{B}|$ is NOT possible

D: In vacuum, no sources, $\vec{B}$ static

$$\rightarrow \operatorname{div} \vec{B} = \operatorname{rot} \vec{B} = 0 \rightarrow \nabla^2 \vec{B} = 0$$

Assume $|\vec{B}(\vec{r})|$ is MAX in $\vec{r} = 0$ and consider a small sphere around this point

$$\vec{B}(\vec{r}) = \vec{B}(0) + \delta\vec{B}(\vec{r})$$

$$|\vec{B}(\vec{r})|^2 = |\vec{B}(0)|^2 + |\delta\vec{B}(\vec{r})|^2 + 2\,\vec{B}(0)\cdot\delta\vec{B}(\vec{r})$$

Without loss of generality $\vec{B}(0) = B_0\,\hat{z}$:

$$|\vec{B}(\vec{r})|^2 = B_0^2 + |\delta\vec{B}|^2 + 2B_0\,\delta B_z(\vec{r})$$

δB_z is also an harmonic function $\nabla^2 \delta B_z = 0$ and such that $\delta B_z(0) = 0$:

$$\delta B_z(\vec{r}) = \sum_{l\geq 1} \sum_m c_{lm}\, r^l\, Y_{lm}(\Omega)$$

$$c_{0,0} = 0$$

For $|\vec{B}|^2$ to be a maximum, $\delta B_z(\vec{r})$ must be negative on all points of sphere, thus its surface integral on the sphere is negative.

This is impossible :

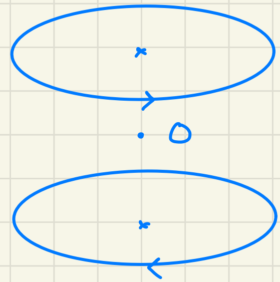
$$\int_{\text{Sphere}} \delta B_z(\vec{r}) = c_{0,0}\cdot 4\pi R^2 = 0$$

Only a minimum of $|\vec{B}|$ is possible in a region void of sources

- Quadrupole trap

Easiest trap configuration

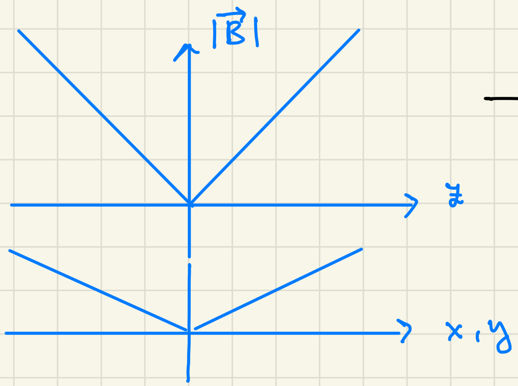
O midpoint between coaxial coils

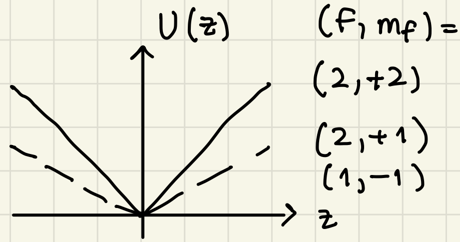


For small displacements around O

$$\vec{B}(\vec{r}) \simeq b\,(3z\hat{z} - \vec{r}), \quad b>0$$

$$|\vec{B}(\vec{r})| = b\sqrt{4z^2 - x^2 - y^2}$$





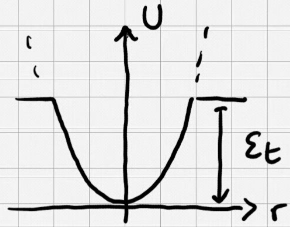
other (f, m_f) are NOT trapped

Quadrupole trap has a relevant drawback : $|\vec{B}(0)| = 0$

At minimum adiabatic approximation breaks, since energy separations vanish.

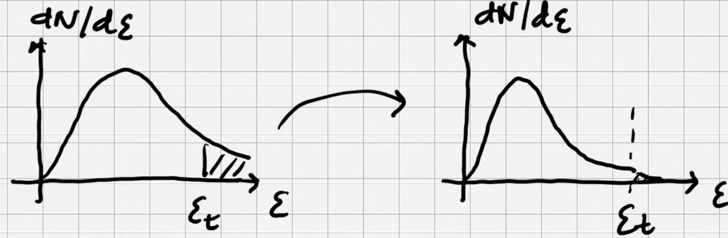
Breaking of adiabatic approximation implies transitions from trapped "low-field seeking" states to untrapped "high-field seeking" states, hence trap loss of atoms.

Evaporation cartoon



Like in a cup of tea, the potential is bound

Atoms with energy > ε_t can escape from trap



Thermalization, by virtue of interatomic collisions, re-populates the tail of the distribution. Equilibrium is reached only when all atoms have left but the time to equilibrium gets longer and longer.

The trimmed distribution contain less atoms at lower temperature
→ in proper circumstances the PSD is higher

We are going to see in what circumstances.

Evaporative cooling

Mean occupation of a state of energy ε

$$n(\varepsilon) = \frac{1}{z^{-1}\exp(\beta\varepsilon) \pm 1} \qquad \beta = \frac{1}{k_B T}$$

$$\simeq z\exp(-\beta\varepsilon) \quad \text{for most evaporation}$$

z is fugacity $\equiv e^{-\beta\mu}$ $\quad \mu$ chemical potential

– Result not proven here:
at high temperature $\beta\mu \gg 1$, $z \ll 1$
and
$$z \simeq n(0)\,\lambda_{dB}^3$$ also known as "phase-space density"

Degeneracy for $z \sim 1$, but MOT $z \sim 10^{-6}$

(1) Number of atoms $\quad N = \int d\varepsilon \, \underbrace{g(\varepsilon)}_{\text{density of states}} n(\varepsilon) =$

$g(\varepsilon)\,d\varepsilon = \#$ states

$$= z\int d\varepsilon \, e^{-\beta\varepsilon} g(\varepsilon) = z\,\underbrace{\zeta(T)}_{\text{partition fn}}$$

$$= n(\vec{0})\,\lambda_{dB}^3\,\zeta(T) = n(\vec{0})\,\underbrace{V_{eff}(T)}_{\text{effective volume}}$$

(2) Total energy $\quad E = \int \varepsilon\, g(\varepsilon)\, n(\varepsilon)\, d\varepsilon$

Using semiclassical approximation, it is easy to calculate $g(\varepsilon)$ for the class of power-law potentials

$$U(r) = B\, r^{a} = U_0 \left(r/R_0\right)^{3/\delta}$$

$\delta = 3/2 \rightarrow U(r)$ harmonic potential
$\delta = 3$ linear potential
$\delta \rightarrow 0$ box potential

$$g(\varepsilon) = A\, \varepsilon^{\delta + 1/2}$$

$$\zeta(T) = A\,(k_B T)^{\delta + 3/2}\, \Gamma(\delta + 3/2)$$

$$V_{eff}(T) = A\,(k_B T)^{\delta + 3/2}\, \Gamma(\delta + 3/2)\, \lambda_{dB}^{3} \sim T^{\delta}$$

with $\Gamma(a) \equiv \int_0^\infty u^{a-1} e^{-u}\, du$ Gamma function

With these results

$$E = N k_B T\,(\delta + 3/2) = N\, \bar{\varepsilon}$$

Evaporation cut: remove all particles w/ energy above $\varepsilon_t = \eta\, k_B T$

$$N \rightarrow N' = N - dN$$

$$E \rightarrow E' = E - dE$$

$dE \geq dN\, \varepsilon_t$ but we approximate $dE \simeq dN\, \eta\, k_B T$

Using the avg energy $E = N\bar{\varepsilon}$

$$(N - dN)(\bar{\varepsilon} - d\bar{\varepsilon}) = N\bar{\varepsilon} - dN\,\eta\, k_B T$$

$$-N d\bar{\varepsilon} - \bar{\varepsilon}\, dN = -dN\,\eta\, k_B T$$

Divide by $N\bar{\varepsilon}$

$$\frac{d\bar{\varepsilon}}{\bar{\varepsilon}} + \frac{dN}{N} = \frac{\eta}{\delta + 3/2}\,\frac{dN}{N}$$

$$\frac{dT}{T} = \frac{d\bar{\varepsilon}}{\bar{\varepsilon}} = \frac{dN}{N}\underbrace{\left(\frac{\eta}{\delta + 3/2} - 1\right)}_{\equiv\, \alpha}$$

$$-d\log T + \alpha\, d\log N = 0$$

$\rightarrow$ N^{α}/T constant

Phase-space density $\mathrm{PSD} = n(\vec{0})\,\lambda_{dB}^3 =$

$$= \frac{N}{V_{eff}(T)}\,\lambda_{dB}^3$$

$$\sim N\, T^{-(\delta + 3/2)}$$

$$\log \mathrm{PSD} = \log N - (\delta + 3/2)\log T$$

$$d\log \mathrm{PSD} = d\log N - (\delta + 3/2)\,\alpha\, d\log N$$

$$= d\log N\left[-\eta + \delta + 5/2\right]$$

$$\rightarrow \quad \frac{\mathrm{PSD}'}{\mathrm{PSD}} = \left(\frac{N'}{N}\right)^{[\dots]}$$

The PSD increases when N decreases for

$$-\eta + \delta + 5/2 < 0$$

$\eta > \delta + 5/2$ i.e. the energy cut should be large enough

linear pot. $\delta = 3 \rightarrow \eta > 11/2$

harm. $\delta = 2$ $\eta > 4$

box $\delta = 0$ $\eta > 5/2$

Larger η, more efficient evaporation but also slower

typical $\eta = 6.5, \eta = 2$

$-6.5 + 2 + 2.5 = -2$

– Collision rate $\gamma = n(0) v \sigma \sim \dfrac{N}{V_{eff}(T)} T^{1/2} \sim N T^{1/2 - \delta}$

$$d \log \gamma = d \log N + \left(\tfrac{1}{2} - \delta\right) \alpha \, d \log N$$

$$= d \log N \left[1 + \frac{1/2 - \delta}{\delta + 3/2} (\eta - \delta - 3/2) \right]$$

Run-away $\delta + \frac{3}{2} + \left(\frac{1}{2} - \delta\right)\left(\eta - \left(\delta + \frac{3}{2}\right)\right) < 0$

$$\left(\delta + \tfrac{3}{2}\right)\left(1 - \tfrac{1}{2} + \delta\right) + \eta\left(\tfrac{1}{2} - \delta\right) < 0$$

$$\eta > \frac{(\delta + 3/2)(\delta + 1/2)}{(\delta - 1/2)}$$

lin $\delta = 3$ $\quad \eta > \dfrac{\frac{9}{2} \cdot \frac{7}{2}}{\frac{5}{2}} = \dfrac{63}{10}$

har. $\delta = 3/2$ $\quad \eta > \dfrac{3 \cdot 2}{1} = 6$

box $\delta = 0$ $\quad \eta > \dfrac{\frac{3}{2} \cdot \frac{1}{2}}{-1/2} = -\frac{3}{2}$ always (!)