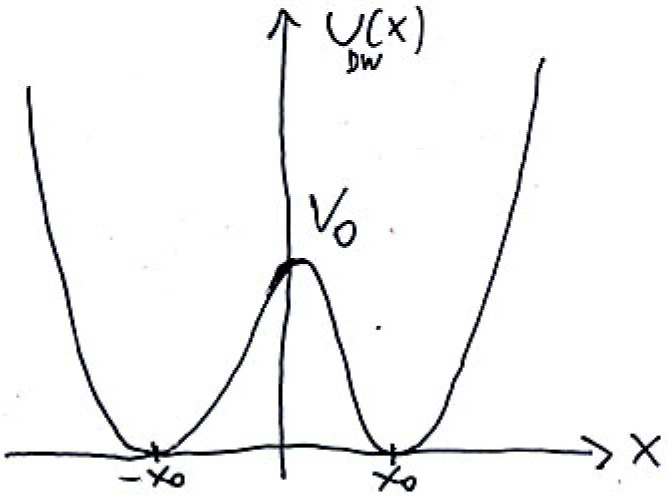
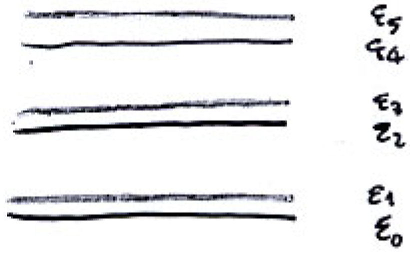
# JOSEPHSON EFFECT WITH BEC




SYMMETRIC DOUBLE-WELL POTENTIAL




ITS ENERGY LEVELS

WEAK LINK : VERY HIGH BARRIER

ONLY THE LOWEST DOUBLET IS IMPORTANT

$$\begin{cases} \varphi_L(x) = \frac{1}{\sqrt{2}}\left(\Phi_0(x) - \Phi_1(x)\right) \\ \varphi_R(x) = \frac{1}{\sqrt{2}}\left(\Phi_0(x) + \Phi_1(x)\right) \end{cases}$$

WAVE FUNCTION LOCALIZED ON THE LEFT WELL

WAVE FUNCTION LOCALIZED ON THE RIGHT WELL

$$i\hbar \frac{\partial}{\partial t} f(x,t) = \left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + U_{DW}(x) + g\,|f(x,t)|^2\right] f(x,t)$$

1D GPE

$$f(x,t) = f_L(t)\,\varphi_L(x) + f_R(t)\,\varphi_R(x)$$

TWO-MODE APPROXIMATION

$$\begin{cases} f_L(t) = \sqrt{N_L(t)}\, e^{i\theta_L(t)} \\ f_R(t) = \sqrt{N_R(t)}\, e^{i\theta_R(t)} \end{cases}$$

$$N = N_L(t) + N_R(t)$$

INSERTING THE TWO-MODE WAVE FUNCTION IN THE 1D GPE AND SETTING

$$\begin{cases} z(t) = \dfrac{N_L(t) - N_R(t)}{N} & \text{FRACTIONAL POPULATION IMBALANCE} \\ \theta(t) = \theta_R(t) - \theta_L(t) & \text{PHASE DIFFERENCE} \end{cases}$$

ONE FINDS :

$$\begin{cases} \dot{z} = -\dfrac{2K}{\hbar}\sqrt{1-z^2}\,\sin(\theta) \\ \dot{\theta} = \dfrac{2K}{\hbar}\dfrac{z}{\sqrt{1-z^2}}\cos(\theta) + \gamma z \end{cases}$$

JOSEPHSON EQUATIONS FOR A BEC

WHERE

$$K = \int_{-\infty}^{+\infty} \varphi_L^*(x)\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_{DW}(x)\right]\varphi_R(x)\,dx$$

TUNNELING ENERGY

$$\gamma = g\int_{-\infty}^{+\infty} \varphi_L(x)^2 \cdot \varphi_R(x)^2\,dx$$

COUPLING STRENGTH

- UNDER THE CONDITION $|z| \ll 1$ THE JOSEPHSON EQUATIONS BECOME

$$\begin{cases} \dot{z} = -\dfrac{2K}{\hbar}\sin(\theta) \\ \dot{\theta} = \dfrac{2K}{\hbar} z\cos(\theta) + \gamma z \end{cases}$$

$$I = -\dot{z} = 2K\sin(\theta) = I_0 \sin(\theta)$$

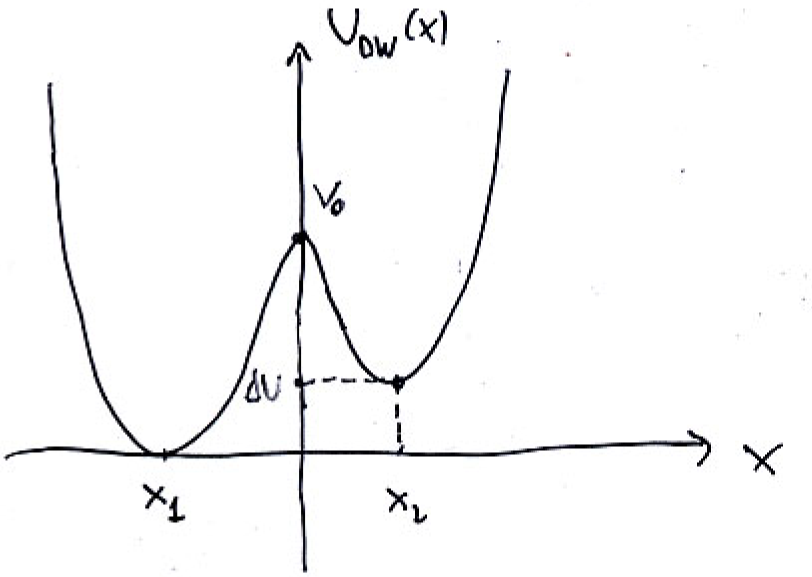
JOSEPHSON CURRENT

AND SETTING $\gamma = 0$ ONE GETS (FOR $\theta \ll 1$) $\ddot{z} + \left(\frac{2K}{\hbar}\right)^2 z = 0$, I.E.

$$\boxed{z(t) = z(0)\cos(\omega_R t)}$$

WITH $\omega_R = \dfrac{2K}{\hbar}$ RABI FREQUENCY

AC JOSEPHSON EFFECT (DUE TO POPULATION IMBALANCE)




$\updownarrow$ $\Delta V$ ENERGY DIFFERENCE

ASYMMETRIC DOUBLE-WELL POTENTIAL

IN THIS CASE THE JOSEPHSON EQUATIONS ARE :

$$\begin{cases} \dot{z} = -\frac{2K}{\hbar}\sqrt{1-z^2}\,\sin(\theta) \\ \dot{\theta} = \frac{2K}{\hbar}\frac{z}{\sqrt{1-z^2}}\cos(\theta) + \frac{\Delta V}{\hbar} + \gamma z \end{cases}$$

COMPLETE JOSEPHSON EQUATIONS FOR A BEC

- UNDER THE CONDITIONS :
  - $|z| \ll 1$
  - $\gamma |z| \ll 1$
  - $\frac{K}{\hbar}|z| \ll 1$

VERY SMALL FRACTIONAL POPULATION IMBALANCE (TYPICAL OF SUPERCONDUCTOR

ONE FINDS :

$$\begin{cases} \dot{z} = -\frac{2K}{\hbar}\sin(\theta) \\ \dot{\theta} = \frac{\Delta U}{\hbar} \end{cases} \iff \begin{cases} I = I_0 \sin(\theta) \\ \dot{\theta} = \frac{\Delta U}{\hbar} \end{cases}$$

THESE ARE THE JOSEPHSON EQUATIONS WHICH APPEAR IN SUPERCONDUCTORS

$$\boxed{I(t) = I_0 \sin\left(\theta_0 + \frac{\Delta U}{\hbar}\cdot t\right)}$$

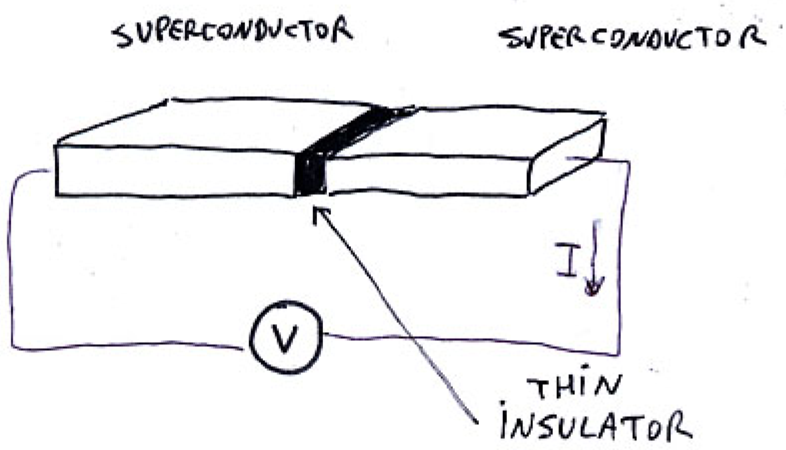
JOSEPHSON CURRENT

$\theta_0$ INITIAL PHASE DIFFERENCE

$\Delta U$ CONSTANT ENERGY DIFFERENCE

- FOR $\Delta U = 0 \Rightarrow$ DC JOSEPHSON EFFECT
- FOR $\Delta U \neq 0 \Rightarrow$ AC JOSEPHSON EFFECT

# JOSEPHSON EFFECT WITH SUPERCONDUCTORS




JOSEPHSON JUNCTION

BRIAN JOSEPHSON PREDICTED IN 1962 THAT

$$I = I_c \sin(\varphi)$$

$$\frac{d\varphi}{dt} = \frac{2e}{\hbar} V$$

JOSEPHSON EQUATIONS
(NOBEL PRIZE 1973)

WHERE $\varphi(t) = \varphi_L(t) - \varphi_R(t)$ IS THE "PHASE DIFFERENCE"

$V(t)$ IS THE EXTERNAL POTENTIAL

$I(t)$ IS THE ELECTRIC CURRENT

$I_c$ IS THE CRITICAL CURRENT

<u>REMARKABLY</u>:

- $V(t) = 0 \implies I(t) = I_0 \sin(\varphi(0))$ CONSTANT (DC) CURRENT
- $V(t) = V_0 \implies I(t) = I_0 \sin\left(\frac{2e}{\hbar} V_0 t + \varphi(0)\right)$ ALTERNATE CURRENT (AC)

THE JOSEPHSON EFFECT CAN BE EXPLAINED AS DUE TO THE TUNNELING OF A GAS OF "COOPER PAIRS" ( PAIRS OF ELECTRONS WITH TOTAL SPIN ZERO).

THE GAS OF COOPER PAIRS BEHAVES "AS" A BOSE-EINSTEIN CONDENSATE WITH A MACROSCOPIC WAVE FUNCTION

$$\Psi(\vec{r},t) = \underbrace{\sqrt{N_L(t)}\, e^{i\theta_L(t)}}_{f_L(t)} \varphi_L(\vec{r}) + \underbrace{\sqrt{N_R(t)}\, e^{i\theta_R(t)}}_{f_R(t)} \varphi_R(\vec{r})$$

WHERE :

$$\begin{cases} i\hbar \frac{\partial}{\partial t} f_L = U_L f_L + K f_R \\ i\hbar \frac{\partial}{\partial t} f_R = U_R f_R + K f_L \end{cases}$$

WITH

$$U_L = \frac{qV}{2} = \frac{2eV}{2} = eV$$

$$U_R = -\frac{qV}{2} = -\frac{2eV}{2} = -eV$$

$$\Rightarrow U_L - U_R = 2eV$$

AND $K$ COUPLING ENERGY ( TUNNELING ENERGY )

FROM THESE EQUATIONS ONE EASILY FINDS THE JOSEPHSON EQUATION WITH

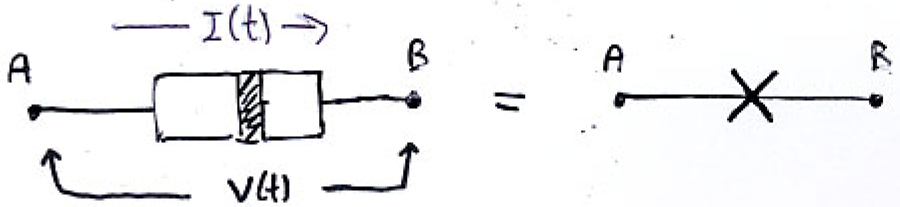
$$I_c = \frac{2}{\hbar} K e \sqrt{N_L(t) N_R(t)} \simeq \frac{2}{\hbar} K e N .$$

AND

$$I = e\dot{N}_L = e\frac{d}{dt}N_L(t) = -e\frac{d}{dt}N_R(t) .$$

# ELECTRONICS WITH JOSEPHSON JUNCTION

A JOSEPHSON JUNCTION CAN BE CONSIDERED AS A BIPOLAR ELEMENT



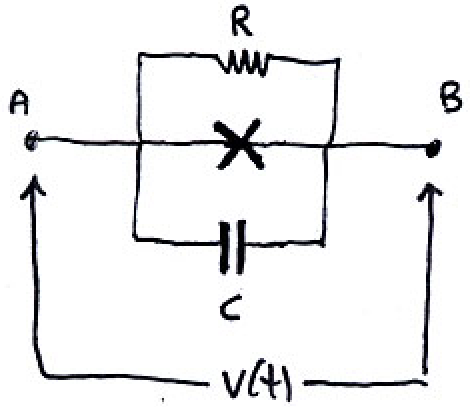

SUCH THAT

$$I = I_c \sin(\varphi) = I_c \sin\left( \int_0^t \frac{2e}{\hbar} V(t')\, dt' + \varphi(0) \right)$$

BECAUSE

$$\dot{\varphi} = \frac{d\varphi}{dt} = \frac{2e}{\hbar} V \quad \Rightarrow \quad \varphi(t) = \varphi(0) + \frac{2e}{\hbar} \int_0^t V(t')\, dt' .$$

REAL JOSEPHSON JUNCTIONS HAVE RESISTIVE AND CAPACITIVE EFFECTS WHICH CAN BE MODELLED AS




RCSJ MODEL

$$I = I_{Jos} + I_{Res} + I_{Cap}$$

$$I_{Jos} = I_c \sin(\varphi)$$

$$I_{Res} = \frac{V}{R} = \frac{1}{R} \frac{\hbar}{2e} \dot{\varphi}$$

$$I_{Cap} = C \frac{d}{dt} V = C \frac{\hbar}{2e} \ddot{\varphi}$$

NAMELY :

$$I = I_c \sin(\varphi) + \frac{\hbar}{2eR} \dot{\varphi} + \frac{\hbar C}{2e} \ddot{\varphi}$$

OR EQUIVALENTLY :

$$\boxed{M \ddot{\varphi} + \eta M \dot{\varphi} + \frac{\partial U}{\partial \varphi} = 0}$$

$$\begin{cases} M = C\left(\frac{\hbar}{2e}\right)^2 \\ U(\varphi) = -E_J \cos(\varphi) - \gamma \varphi . \\ E_J = \hbar I_c / (2e) \\ \gamma = \hbar I / (2e) \end{cases}$$