Quantum Effects in Josephson Junctions - Part 2

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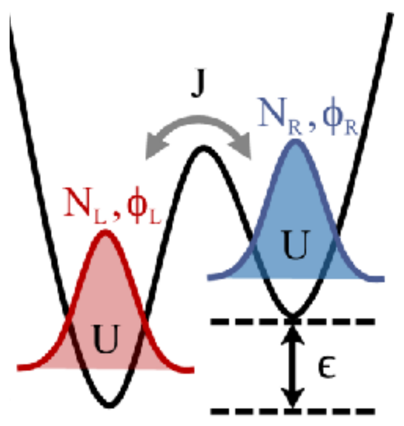
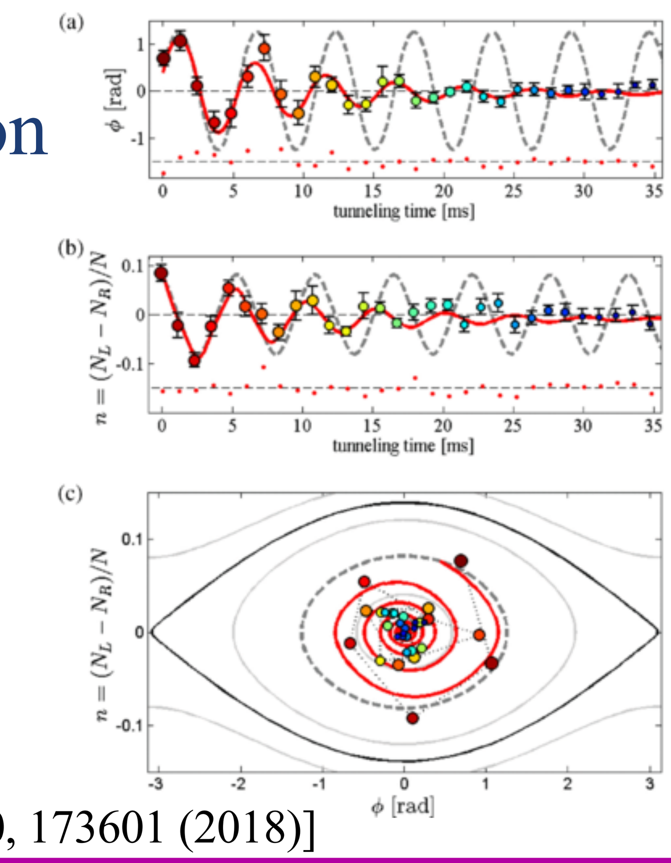
Introduction

✓ Bose Josephson junction

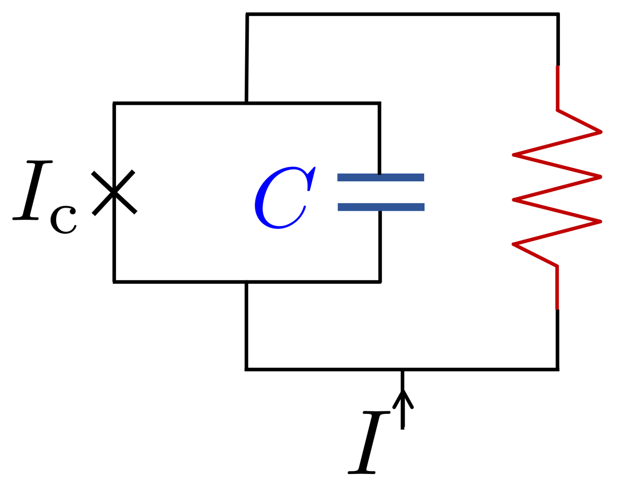


$$\Omega = \frac{\sqrt{J(UN+J)}}{\hbar}$$

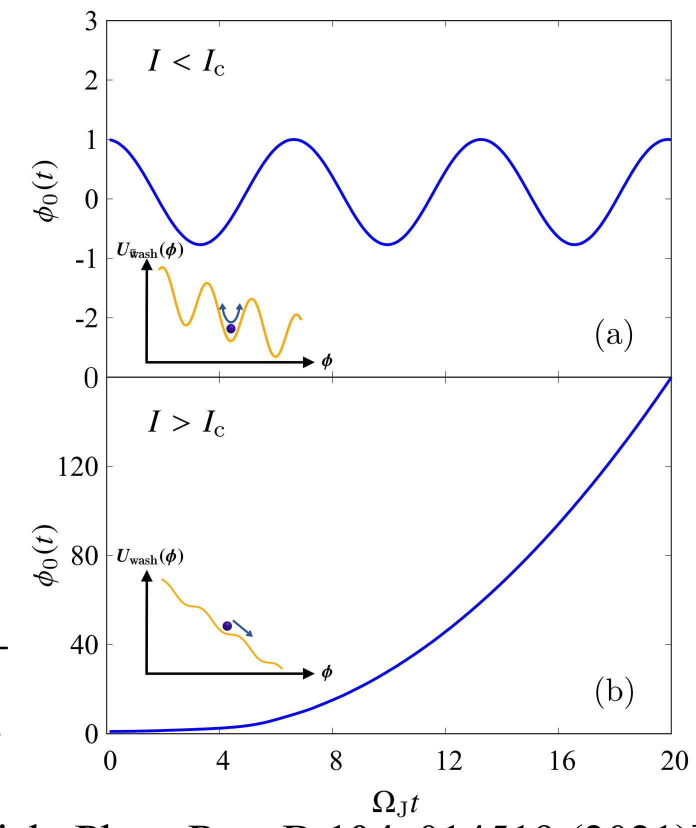


[M. Pigneur *et al.*, Phys. Rev. Lett. **120**, 173601 (2018)]

✓ Resistively & capacitively shunted Josephson circuit



$$\Omega = \frac{1}{\hbar}\sqrt{\frac{2e\hbar I_c}{C}}$$



[KF and L. Salasnich, Phys. Rev. B **104**, 014519 (2021)]

Our work

We find the leading quantum correction to Josephson frequency in Josephson junctions.

$$\tilde{\Omega} = \Omega + (\text{Quantum correction})$$

Bose Josephson junction

$$L = \sum_{j=1,2}\left[i\hbar\psi_j^*\partial_t\psi_j - \frac{U}{2}|\psi_j|^4\right] + \frac{J}{2}\left[\psi_1^*\psi_2 + \psi_2^*\psi_1\right]$$

U : onsite interaction strength J : tunnel coupling

$\psi_j(t) = \sqrt{N_j(t)}e^{i\phi_j(t)}$: complex bosonic field on a site j at a real-time t

$N_j(t)$: number of bosons on a site j at a real-time t

$\phi_j(t)$: phase angle on a site j at a real-time t

Total number & total phase $N(t) = N_1(t) + N_2(t)$ $\bar{\phi}(t) = \phi_1(t) + \phi_2(t)$

Population imbalance & relative phase $z(t) = \frac{N_1(t) - N_2(t)}{N(t)}$ $\phi(t) = \phi_2(t) - \phi_1(t)$

Integrating out $\bar{\phi}$ and N

$$L_{\rm rel}[\phi, z] = \frac{N\hbar}{2}z\dot{\phi} - \frac{UN^2}{4}z^2 + \frac{JN}{2}\sqrt{1-z^2}\cos\phi$$

▸ EoM

$$\begin{cases}\ddot{\phi} + \Omega^2\phi = 0\\ \ddot{z} + \Omega^2 z = 0\end{cases}$$

▸ Josephson frequency

$$\Omega = \frac{\sqrt{J(UN+J)}}{\hbar} = \begin{cases}\Omega_R = \frac{J}{\hbar} & (UN/J \ll 1)\\ \Omega_J = \frac{\sqrt{UNJ}}{\hbar} & (UN/J \gg 1)\end{cases}$$

▸ Path-integral propagator

$$K(\phi_f, T|\phi_i, 0) = \int_{(\phi_i,0)}^{(\phi_f,T)}\mathcal{D}\phi\int\mathcal{D}z e^{iS_{\rm rel}[\phi,z]/\hbar} = \int_{(\phi_i,0)}^{(\phi_f,T)}\mathcal{D}\phi e^{iS_0[\phi]/\hbar}$$

▸ Phase action

$$S_0[\phi] = \int dt\left[\frac{N\hbar^2\dot{\phi}^2}{4(UN + J\cos\phi)} + \frac{JN}{2}\cos\phi\right]$$

$$\simeq S_J[\phi] = \int dt\left[\frac{M_J}{2}\dot{\phi}^2 - V(\phi)\right]$$

in the Josephson regime $UN/J \gg 1$

$$M_J \equiv \frac{\hbar^2}{2U} \qquad V(\phi) \equiv \frac{JN}{2}(1-\cos\phi)$$

$$\lambda \equiv -\frac{JN}{48} \qquad = \frac{M_J\Omega_J^2}{2}\phi^2 + U(\phi) = \frac{M_J\Omega_J^2}{2}\phi^2 + \lambda\phi^4 + \mathcal{O}(\phi^6)$$

Quantum fluctuations

Effective action

$$\Gamma_J[\phi] = \int dt\left[\frac{Z(\phi)}{2}\dot{\phi}^2 - V_{\rm eff}(\phi)\right]$$

up to the **2nd order derivative expansion**

▸ Validity conditions

$$|\phi| \ll 1, \quad 1 \ll \frac{UN}{J} \ll N^2$$

▸ $\hbar$ expansion

$$Z(\phi) = M_J + Z_1(\phi) + Z_2(\phi) + \cdots = M_J + Z_1(\phi) + \mathcal{O}(\hbar^2)$$

$$V_{\rm eff}(\phi) = \frac{M_J\Omega_J^2}{2}\phi^2 + V_{\rm eff}^{(1)}(\phi) + V_{\rm eff}^{(2)}(\phi) + \cdots$$

$$= \frac{M_J\Omega_J^2}{2}\phi^2 + V_{\rm eff}^{(1)}(\phi) + \mathcal{O}(\hbar^2)$$

$$Z_1(\phi) = \frac{\hbar}{32M_J^2}\frac{\left[\partial_\phi^3 U(\phi)\right]^2}{\left[\Omega_J^2 + \partial_\phi^2 U(\phi)/M_J\right]^{5/2}} = \mathcal{O}(\phi^2)$$

$$V_{\rm eff}^{(1)}(\phi) = \frac{\hbar}{2}\left[\sqrt{\Omega_J^2 + \frac{1}{M_J}\partial_\phi^2 U(\phi)} - \Omega_J\right] = \frac{3\hbar\lambda}{M_J\Omega_J}\phi^2 + \mathcal{O}(\phi^4)$$

Josephson frequency with 1-loop correction

$$\tilde{\Omega}_J = \Omega_J\sqrt{1 - \frac{1}{2}\sqrt{\frac{U}{JN}}}$$

Alternative approach

EoM: $\ddot{\phi} + \Omega_J^2\sin\phi = 0$

$$\phi(t) = \phi_0(t) + \tilde{\phi}(t) \qquad \langle\tilde{\phi}\rangle = 0 \qquad \langle\tilde{\phi}^2\rangle = \sqrt{\frac{U}{JN}}$$

Mean-field configuration | Quantum fluctuations

$$\ddot{\phi}_0 + \Omega_J^2\left(1 - \frac{1}{2}\langle\tilde{\phi}^2\rangle\right)\sin\phi_0 = 0$$

$$= \tilde{\Omega}_J^2$$

· · · *consistent with the result of effective action formalism*

Summary

[KF, J. Tempere, and L. Salasnich, Phys. Rev. B **105**, 134510 (2022)]

- We have obtained a leading quantum correction to the Josephson frequency in a 1D Josephson junction via effective action formalism.
- Our result has been verified also by another approach through the equation of motion.

✓ Bose Josephson junction

[M. Pigneur *et al.*, Phys. Rev. Lett. **120**, 173601 (2018)]

$$\tilde{\Omega}_J = \Omega_J\sqrt{1 - \frac{1}{2}\sqrt{\frac{U}{JN}}} \qquad \frac{\Omega_J - \tilde{\Omega}_J}{\Omega_J} \simeq 0.1\% \quad \text{with} \quad N = 2500 \quad \frac{UN}{2J} \sim 10^2$$

✓ RCSJ junction

[M. H. Devoret, J. M. Martinis, and J. Clarke, Phys. Rev. Lett. **55**, 1908 (1985)]

$$\tilde{\Omega}_J = \frac{1}{\hbar}\sqrt{\frac{2e\hbar I_0}{C}}\sqrt{1 - \frac{1}{2}\sqrt{\frac{2e^3}{\hbar C I_0}}} \qquad \frac{\Omega_J - \tilde{\Omega}_J}{\Omega_J} \simeq 0.03\% \quad \text{with} \quad \sqrt{\frac{8e^3}{\hbar C I_0}} \simeq 2.3\times 10^{-3}$$