

# Quantum Effects in Josephson Junctions - Part 1

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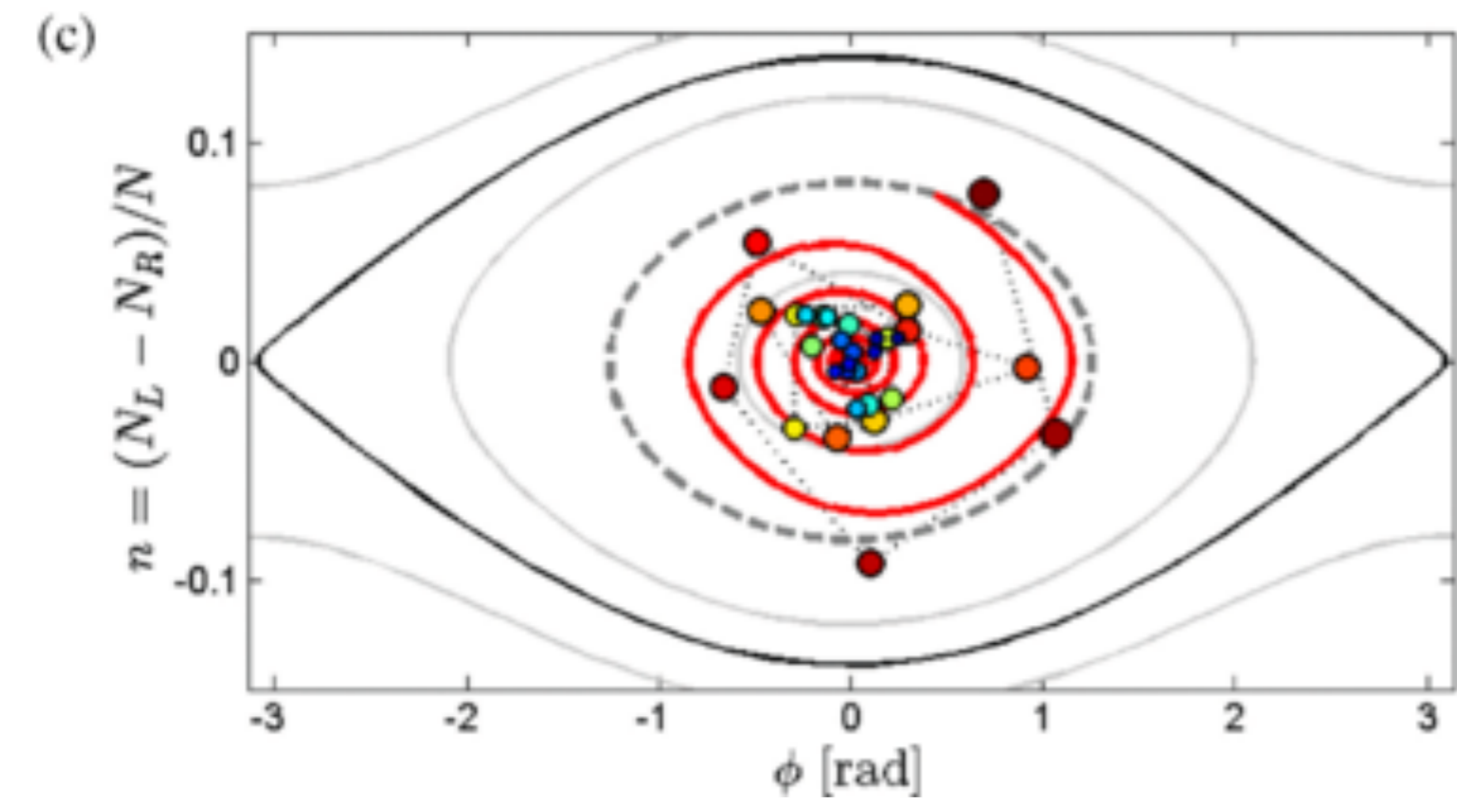
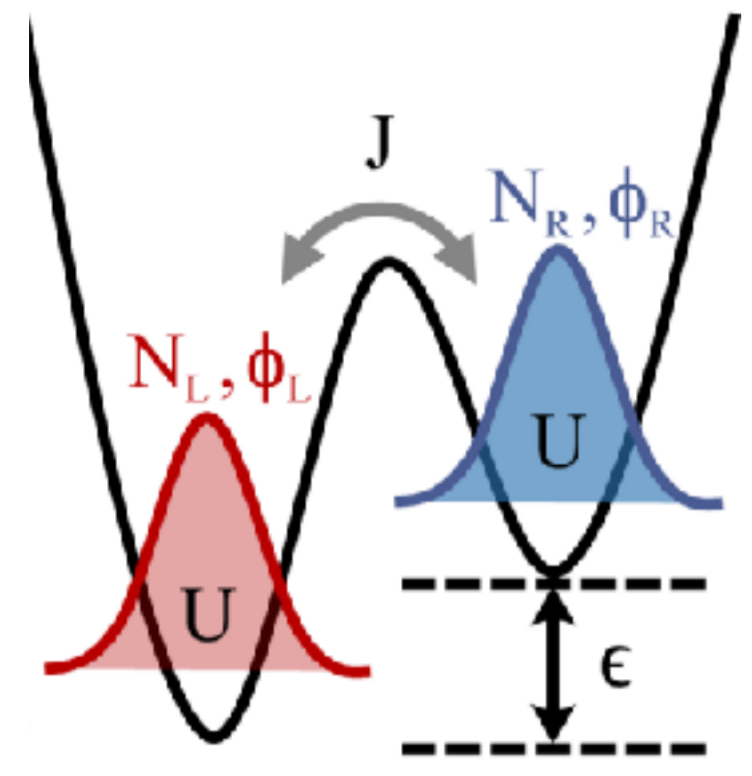
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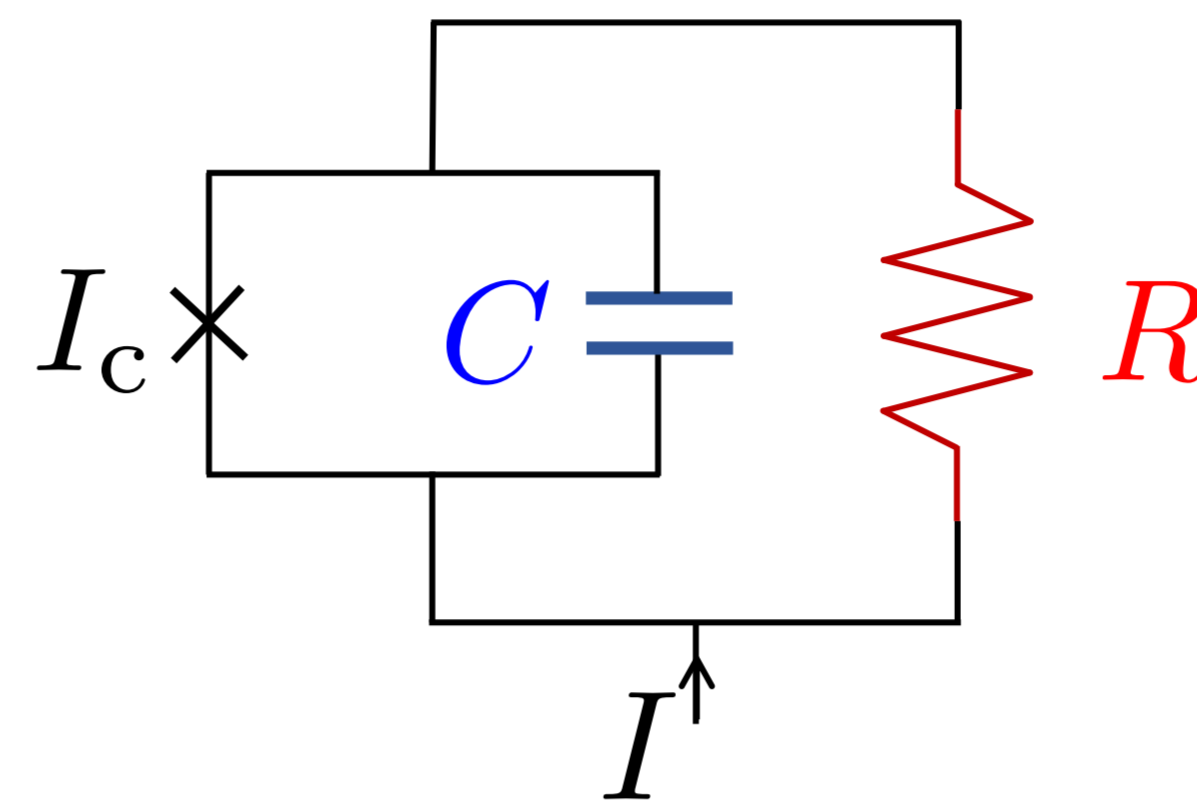
## Introduction

✓ Damped dynamics in a Bose Josephson junction

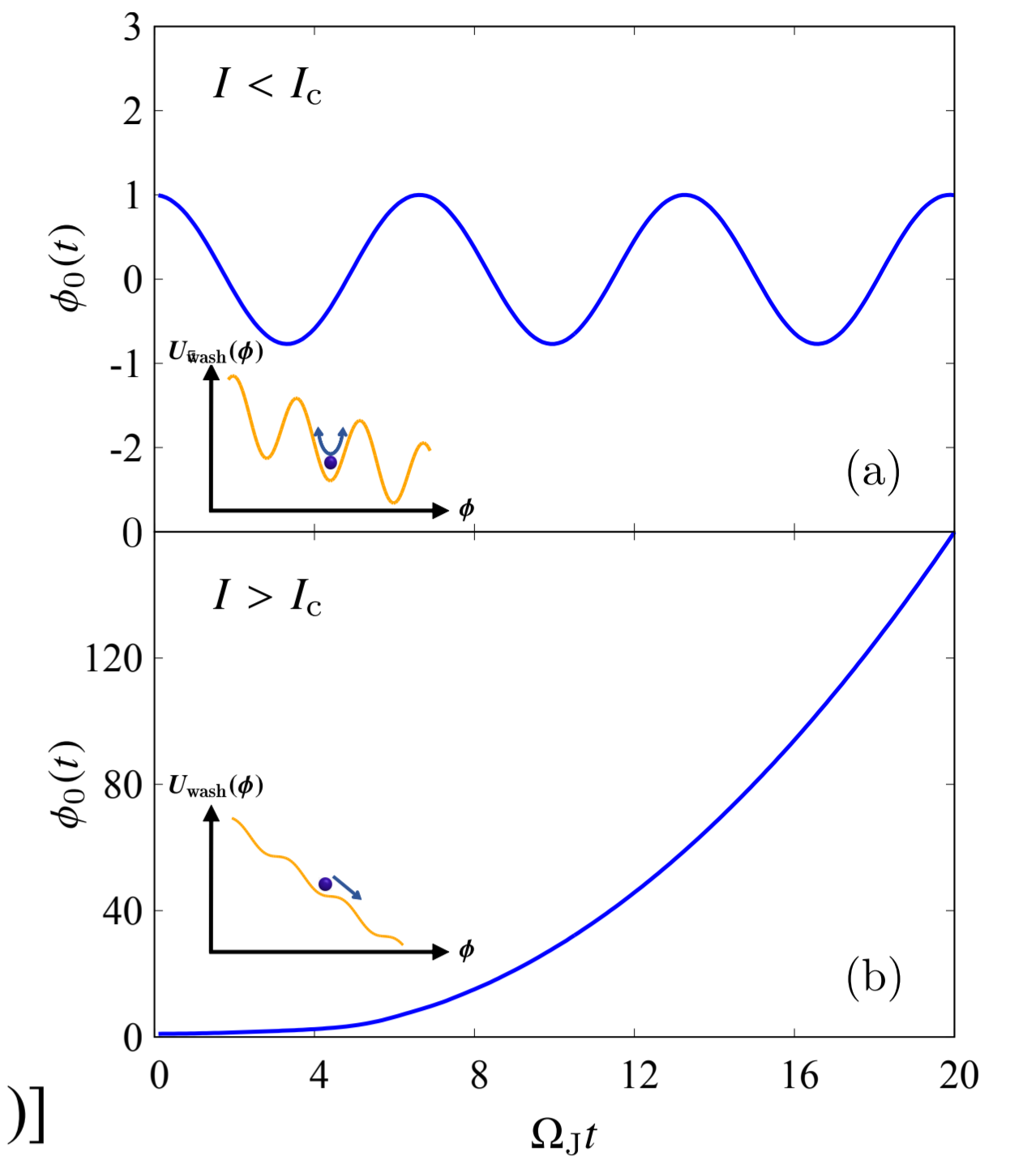


[M. Pigneur *et al.*, Phys. Rev. Lett. **120**, 173601 (2018)]

✓ Resistively & capacitively shunted Josephson circuit



[KF and L. Salasnich, Phys. Rev. B **104**, 014519 (2021)]



## Our work

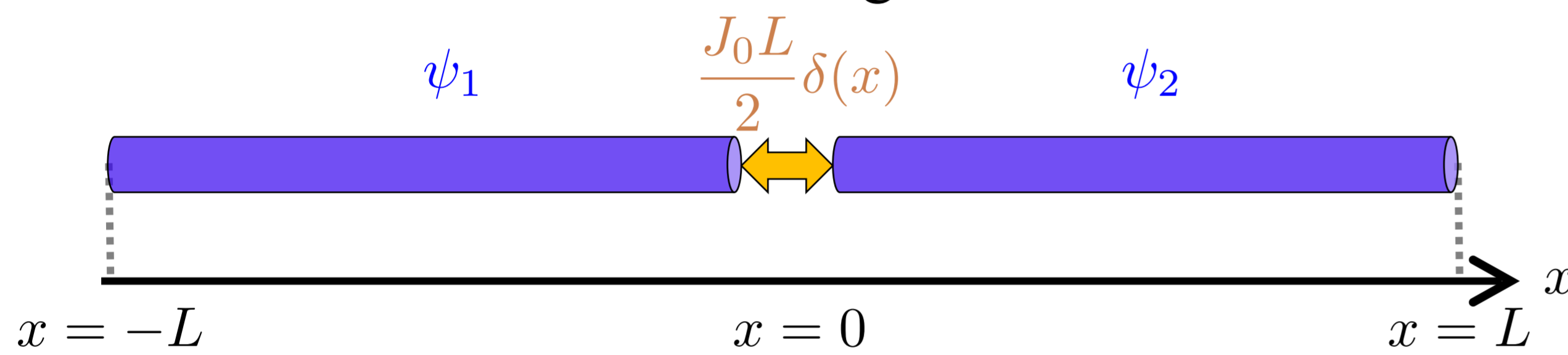
*We clarify the damped dynamics of correlation functions in Josephson junctions subject to quantum noise.*

## 1D Bose Josephson junction

[F. Binanti, KF, and L. Salasnich, Phys. Rev. A **103**, 063309 (2021)]

$$\mathcal{L} = \sum_{j=1,2} \left[ i\hbar\psi_j^* \partial_t \psi_j - \frac{\hbar^2}{2m} |\partial_x \psi_j|^2 - \frac{g}{2} |\psi_j|^4 \right] + \frac{J(x)}{2} [\psi_1^* \psi_2 + \psi_2^* \psi_1]$$

$J(x) = J_0 L \delta(x)$ : head-to-tail configuration



▶ **Effective Hamiltonian for Josephson mode**

$$\psi_j = \sqrt{\rho_j} e^{i\phi_j} \quad \phi \equiv \phi_1 - \phi_2 \quad \zeta \equiv (\rho_1 - \rho_2)/(2\bar{\rho}) \quad \bar{\rho} \equiv (\rho_1 + \rho_2)/2$$

$$\phi(x, t) = \frac{1}{\sqrt{L}} \sum_{n=0}^{\infty} \Phi_n(x) q_n(t) \quad \phi_0(t) \equiv \phi(0, t) \quad Q_n(t) \equiv q_n(t)$$

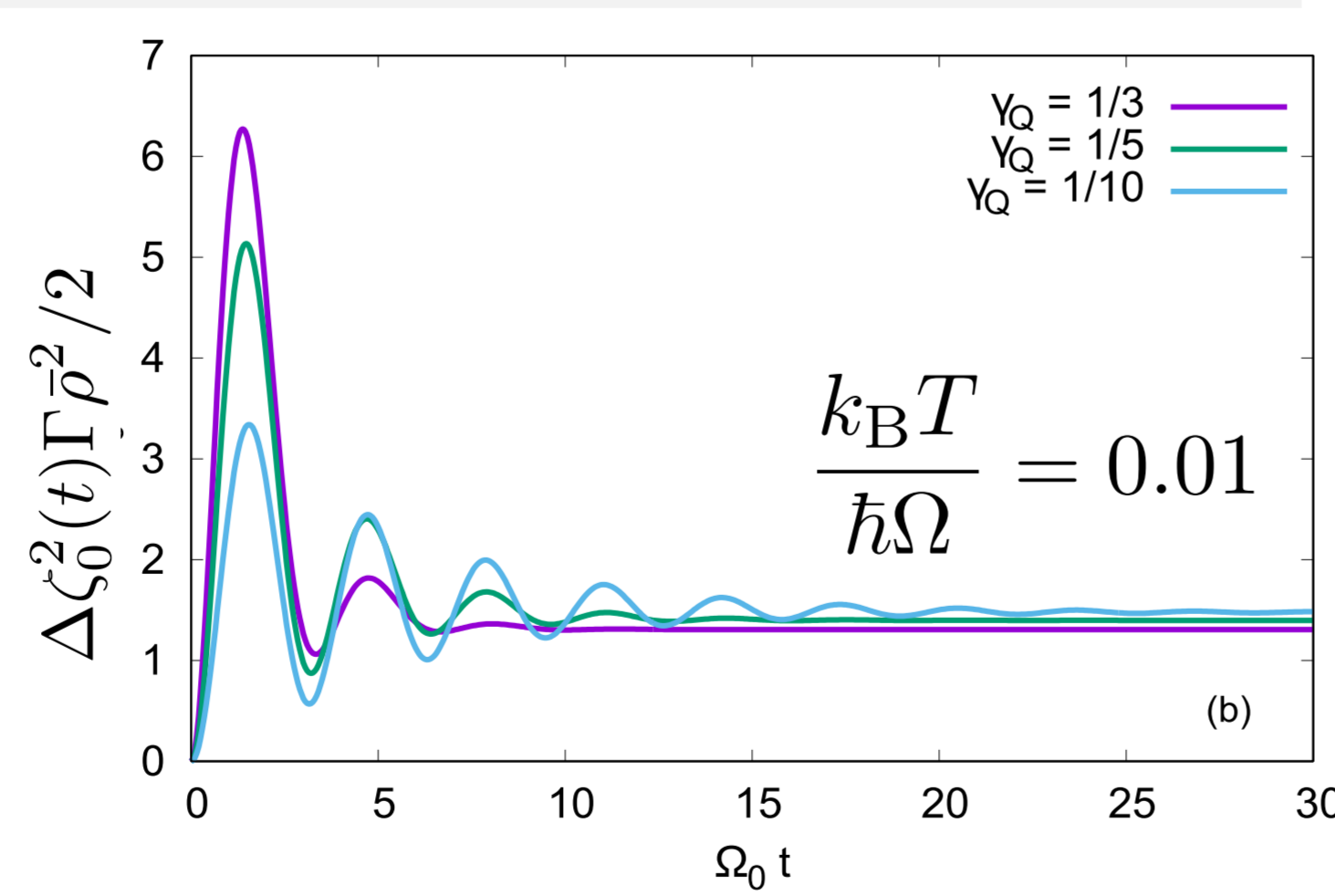
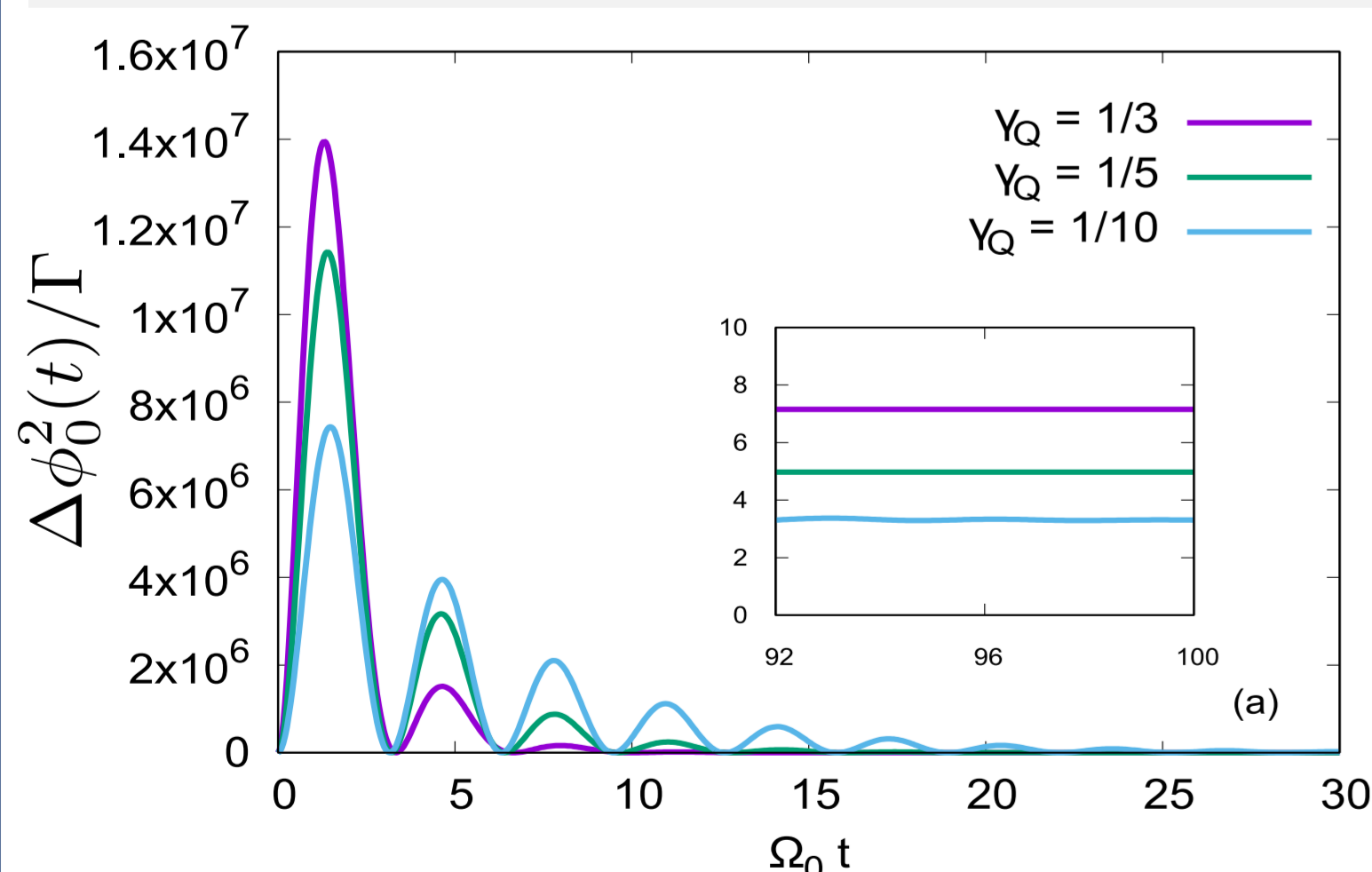
$$H = \frac{P_0^2}{2M} - J_0 L \bar{\rho} \cos \phi_0 + \sum_{n=1}^{\infty} \left[ \frac{(P_n + P_0)^2}{2M} + \frac{M\omega_n^2}{2} Q_n^2 \right]$$

$$\begin{cases} \ddot{\phi}_0 + \gamma \dot{\phi}_0 + \Omega^2 \phi_0 = \xi_\phi \\ \ddot{\zeta}_0 + \gamma \dot{\zeta}_0 + \Omega^2 \zeta_0 = \xi_\zeta \end{cases}$$

Intrinsic coupling with bath modes (Velocity-coupling model)  
**Generalized Langevin equations for Josephson mode**

$$M = \frac{\hbar^2}{2gL} \quad \omega_n = ck_n \quad \langle \{ \xi_\phi(t), \xi_\phi(0) \} \rangle = \frac{2\gamma}{ML^2\Omega^2} \int \frac{d\omega}{2\pi} e^{i\omega t} \hbar\omega^3 \coth\left(\frac{\hbar\omega}{2k_B T}\right)$$

$$c = \sqrt{\frac{g\bar{\rho}}{m}} \quad \omega_{\text{cut}} = c \frac{\pi N}{L} \quad \langle \{ \xi_\zeta(t), \xi_\zeta(0) \} \rangle = \frac{2M\Omega^2\gamma}{\hbar^2\bar{\rho}^2 L^2} \int \frac{d\omega}{2\pi} e^{i\omega t} \hbar\omega \coth\left(\frac{\hbar\omega}{2k_B T}\right)$$



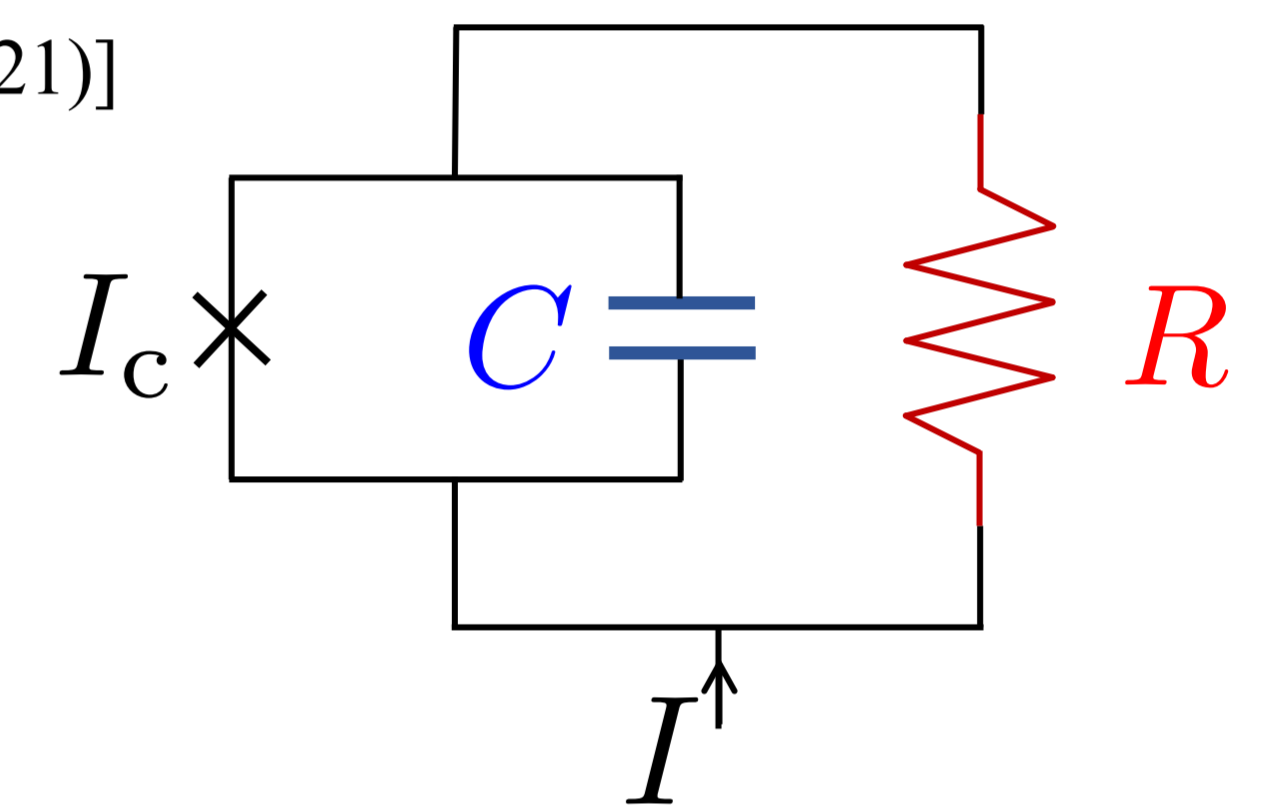
- Asymptotic correlations depend on  $\gamma = \bar{\rho}J_0/(Mc)$ .
- Phase correlation is strongly enhanced.
- In-phase oscillations.

$$\Gamma = \frac{2\hbar}{M\Omega} \quad \gamma_Q \equiv \frac{\gamma}{2\Omega}$$

## RCSJ junction

[KF and L. Salasnich, Phys. Rev. B **104**, 014519 (2021)]

$$\begin{cases} C\dot{V} + \frac{V}{R} + I_c \sin \phi = I + \xi \\ \dot{\phi} = \frac{2e}{\hbar} V \end{cases}$$



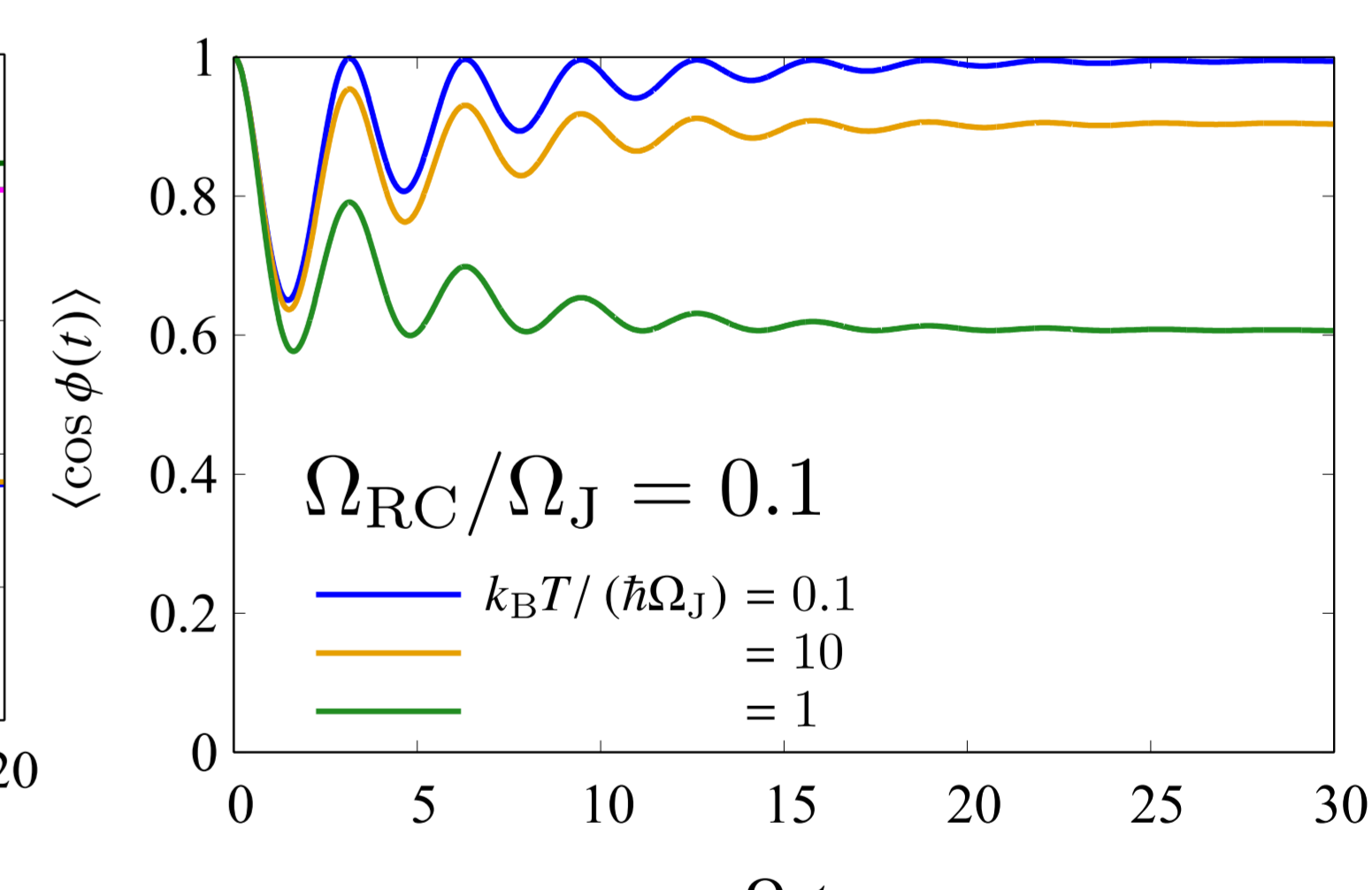
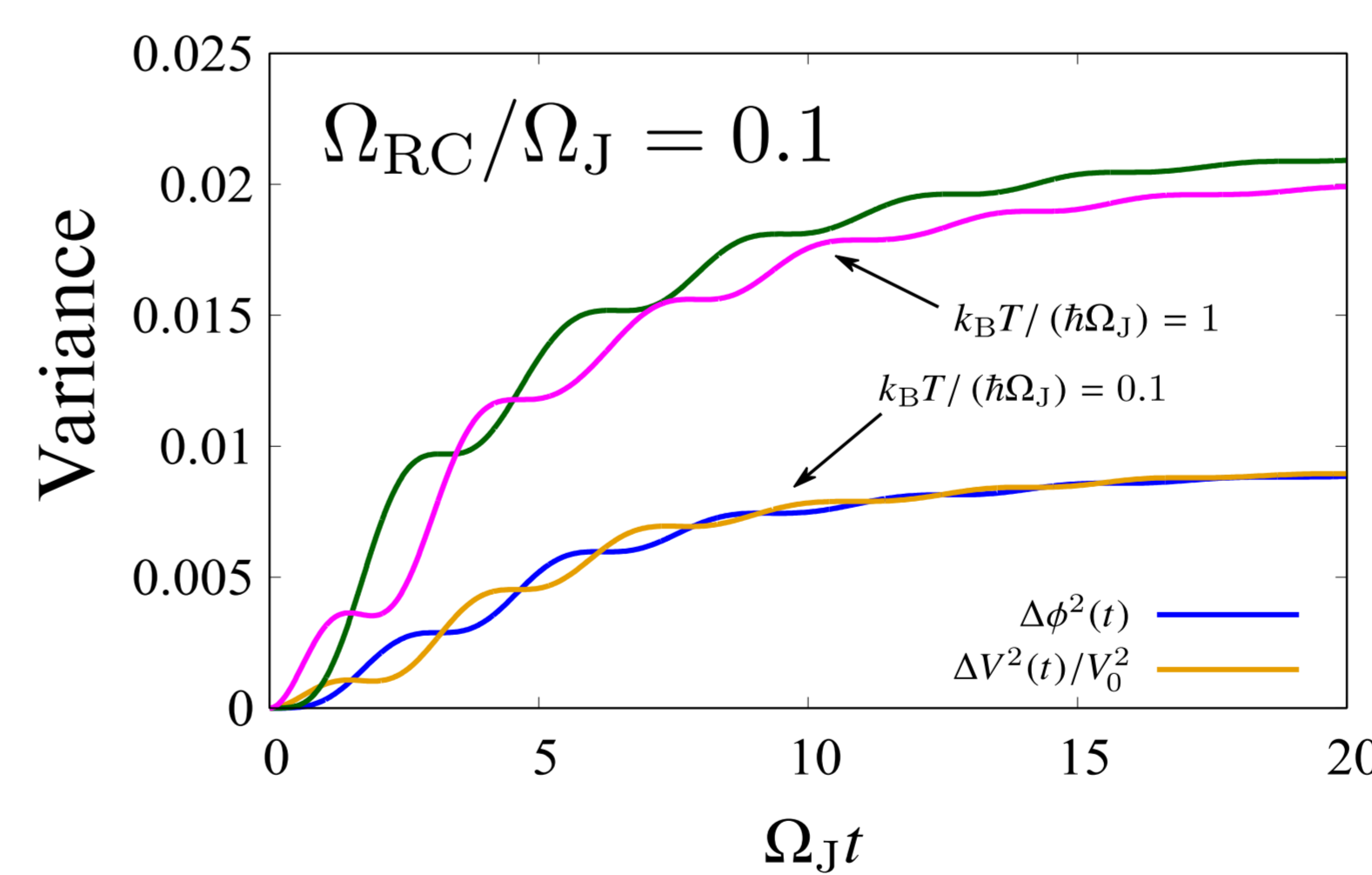
$$\ddot{\phi} + 2\Omega_{RC}\dot{\phi} + \Omega_J^2\phi = \Omega_J^2 \left( \frac{I}{I_c} + \frac{\xi}{I_c} \right)$$

$$\Omega_J = \sqrt{\frac{2eI_c}{\hbar C}} \quad \Omega_{RC} = \frac{1}{2RC}$$

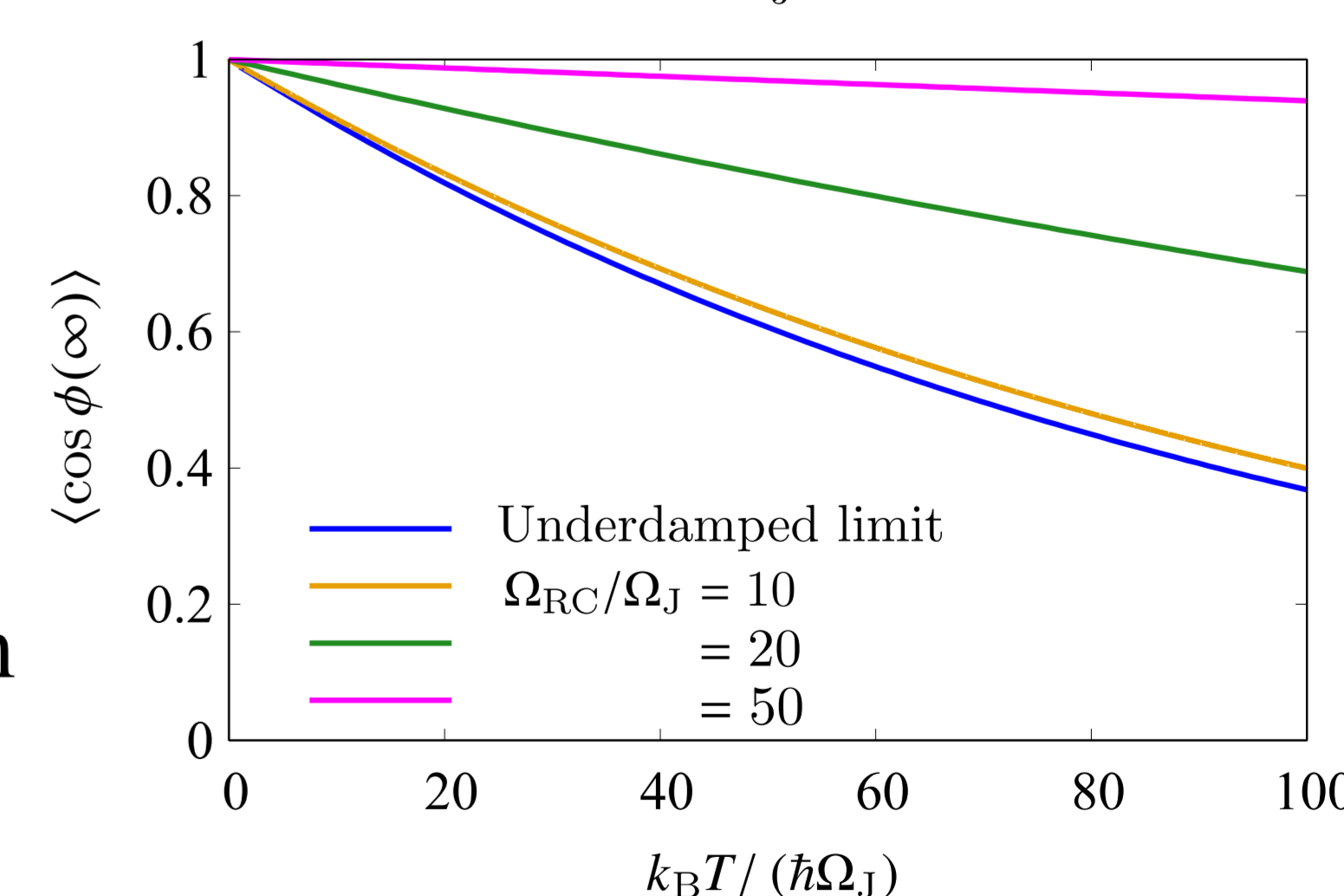
$$\langle \xi(t)\xi(0) \rangle = \frac{2}{R} \int \frac{d\omega}{2\pi} e^{i\omega t} \hbar\omega \coth\left(\frac{\hbar\omega}{2k_B T}\right)$$

$$\frac{2e\Omega_J}{I_c} = 0.01 \quad \omega_{\text{cut}} = \Delta(T)/\hbar \quad \Delta(T): \text{superconducting gap}$$

[M. H. Devoret, J. M. Martinis, and J. Clarke, Phys. Rev. Lett. **55**, 1908 (1985)]



- Out-of-phase oscillations of phase & voltage variances.
- Coherence decay earlier at a higher temperature.
- Coherence decay earlier with smaller damping  $\Omega_{RC}$ .



## Bose Josephson junction

Intrinsic coupling through momenta

- Enhancement of  $\langle \phi_0^2(t) \rangle$
- In-phase oscillations of  $\Delta\phi_0^2(t)$  and  $\Delta\zeta_0^2(t)$
- Correlations in the long-time limit depend on the damping (friction) constant in the quantum regime.

## Summary

## RCSJ junction

Current noise & friction generated by a resistor

- Same order of  $\langle \phi^2(t) \rangle$  and  $\langle V^2(t) \rangle$
- Out-of-phase oscillations of  $\Delta\phi^2(t)$  and  $\Delta V^2(t)$