Dissipative and non-dissipative quantum dynamics of bosons in a Josephson junction

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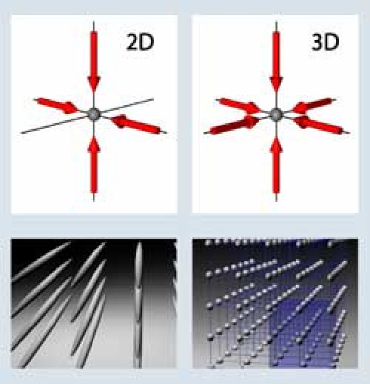
In collaboration with Sandro Wimberger

Summary

- Schrödinger cats in double-well potentials
- Mean-field quantum dynamics of the bosonic Josephson junction
- Recent puzzling experiment: relaxation without dissipation
- Exact quantum dynamics of the bosonic Josephson junction
- Conclusions and open problems

Schrödinger cats in double-well potentials (I)

The study of neutral atoms trapped with light is a very hot topic of research.



Changing the intensity and shape of the external optical lattice, it is now possible to trap atoms in very different configurations. One can have many atoms per site bit also few atoms per site.

Schrödinger cats in double-well potentials (II)

Dilute identical bosonic atoms can be described by a quantum field operator $\hat{\psi}(\mathbf{r}, t)$ which satisfies the Heisenberg equation of motion

$$i\hbar \frac{\partial}{\partial t} \hat{\psi}(\mathbf{r}, t) = \left[-\frac{\hbar^2}{2m} \nabla^2 + U(\mathbf{r}) + \frac{4\pi\hbar^2 a_s}{m} |\hat{\psi}(\mathbf{r}, t)|^2 \right] \hat{\psi}(\mathbf{r}, t) \, , \quad (1)$$

where $U(\mathbf{r})$ is the external potential and a_s is the s-wave scattering length of the inter-atomic potential.
We have theoretically studied static and dynamical properties of ultracold bosonic atoms in the following external potential

$$U(\mathbf{r}) = V_{DW}(x) + \frac{1}{2} m \omega_\perp^2 (y^2 + z^2) \, , \quad (2)$$

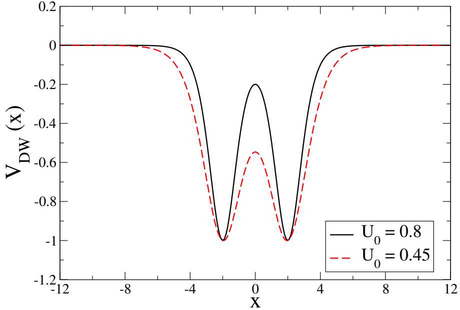
where $V_{DW}(x)$ is a double-well potential, by adopting the following ansatz

$$\hat{\psi}(\mathbf{r}, t) = \Big(\Phi_L(x) \, \hat{a}_L(t) + \Phi_R(x) \, \hat{a}_R(t) \Big) \sqrt{\frac{m\omega_\perp}{\sqrt{\pi}\hbar}} \, e^{-m\omega_\perp (y^2+z^2)/(2\hbar)} \quad (3)$$

where $\hat{a}_j$ and $\hat{a}_j^+$ are respectively the annihilation and creation operators of bosons in the site j ($j = L, R =$Left,Right).

Schrödinger cats in double-well potentials (III)

In this way we have investigated the **macroscopic quantum tunneling** of neutral atoms from one well to the other well. This is the analog of the **Josephson effect of superconductivity** (Josephson junction).



Thus the system is well described by the two-site Bose-Hubbard Hamiltonian

$$\hat{H} = -J\left(\hat{a}_L^+\hat{a}_R + \hat{a}_R^+\hat{a}_L\right) + \frac{U}{2}\left(\hat{N}_L(\hat{N}_L - 1) + \hat{N}_R(\hat{N}_R - 1)\right) \quad (4)$$

where $N_j = \hat{a}_j^+\hat{a}_j$ is the number operator of site j. Here J is the hopping (tunneling) energy and U is the on-site energy.

Schrödinger cats in double-well potentials (IV)

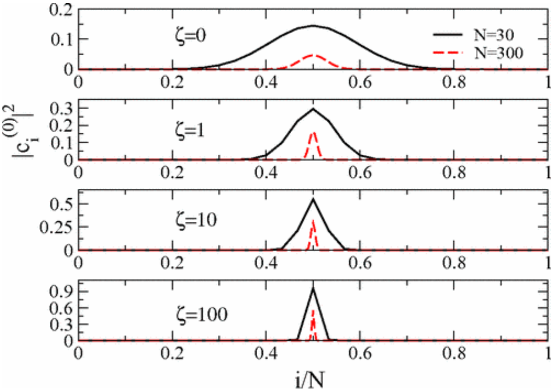
The ground-state of the two-site Bose-Hubbard Hamiltonian with N bosonic atoms

$$|GS\rangle = \sum_{i=0}^{N} c_i^{(0)} \, |i\rangle_L \otimes |N-i\rangle_R \tag{5}$$

is strongly quantum entangled and characterized by the complex coefficients $c_j^{(0)}$ which depend on the ratio U/J. Here $|i\rangle_L$ is the Fock state of i bosons on the site L and $|N-i\rangle_R$ is the Fock state of $(N-i)$ bosons on the site R.

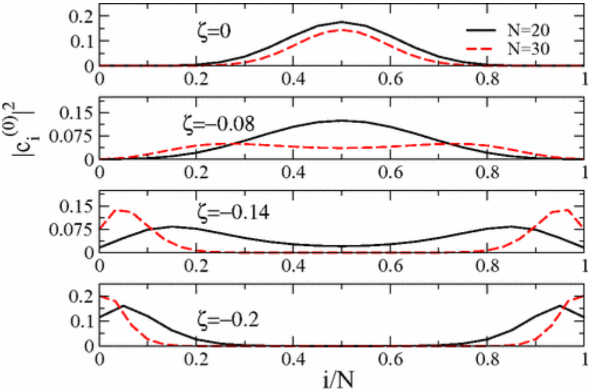
The coefficients $c_j^{(0)}$ are obtained by a direct diagonalization of the $(N+1) \times (N+1)$ Hamiltonian matrix $\hat{H}$.

Schrödinger cats in double-well potentials (V)



Square modulus of the coefficients $c_i^{(0)}$ of the repulsive ($U > 0$) ground state. $|c_i^{(0)}|^2$ gives the probability of finding i bosons on the left well and $N - i$ bosons on the right well. Here $\zeta = U/J$ and N is the total number of bosons. [PRA **83**, 053607 (2011)]

Schrödinger cats in double-well potentials (VI)



Square modulus of the coefficients $c_i^{(0)}$ of the attractive ($U < 0$) ground state. $|c_i^{(0)}|^2$ gives the probability of finding i bosons on the left well and $N - i$ bosons on the right well. Here $\zeta = U/J$ and N is the total number of bosons. [PRA **83**, 053607 (2011)].

Schrödinger cats in double-well potentials (VII)

In particular, for $U = 0$ the ground state $|GS\rangle$ is the atomic coherent state $|ACS\rangle$:

$$|GS\rangle = |ACS\rangle = \frac{1}{\sqrt{N!}} \left[\frac{1}{\sqrt{2}} \left(\hat{a}_L^+ + \hat{a}_R^+ \right) \right]^N |0\rangle_L \otimes |0\rangle_R \, . \qquad (6)$$

Instead, for $U/J \gg 1$ the ground state $|GS\rangle$ becomes is the twin-Fock state $|TF\rangle$:

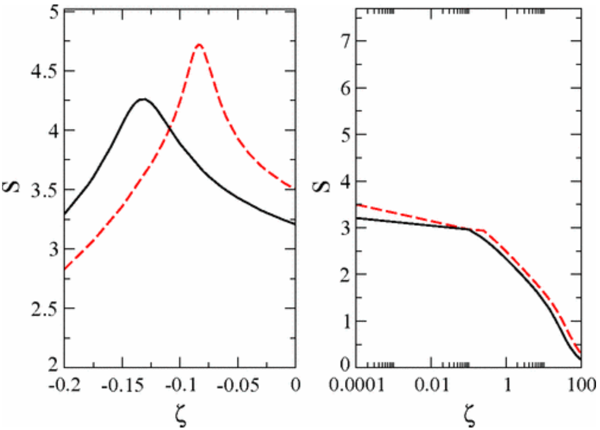
$$|GS\rangle \to |TF\rangle = |\frac{N}{2}\rangle_L \otimes |\frac{N}{2}\rangle_R \, , \qquad (7)$$

while for $U/J \ll -1$ the ground state $|GS\rangle$ becomes the Schrödinger-cat state (NOON state) $|CAT\rangle$:

$$|GS\rangle \to |CAT\rangle = \frac{1}{\sqrt{2}} \left(|N\rangle_L \otimes |0\rangle_R + |0\rangle_L \otimes |N\rangle_R \right) \, . \qquad (8)$$

With ultracold atoms it is quite easy to experimentally obtain $U < 0$ by using the Feshbach-resonance technique. This is instead very difficult with superconducting Josephson junctions.

Schrödinger cats in double-well potentials (VIII)



Entanglement entropy S of the ground-state $|GS\rangle$ of the two-site Bose-Hubbard Hamiltonian as a function of the parameter $\zeta = U/J$. Left panel: attractive bosons ($U < 0$). Right panel: repulsive bosons ($U > 0$). **Solid line**: $N = 20$. Dashed line: $N = 30$. [PRA **83**, 053607 (2011)]

Mean-field dynamics of the Josephson junction (I)

The mean-field quantum dynamics of the bosonic Josephson junction is obtained assuming that the time-dependent many-body state of the system is given by

$$|\psi(t)\rangle = |\alpha_L(t)\rangle \otimes |\alpha_R(t)\rangle \; . \tag{9}$$

where $|\alpha_j(t)\rangle$ is the Glauber coherent state, i.e. the eigenstate of the time-dependent destruction operator $\hat{a}_j(t)$:

$$\hat{a}_j(t)|\alpha_j(t)\rangle = \alpha_j(t)|\alpha_j(t)\rangle \tag{10}$$

with complex eigenvalue $\alpha_j(t) = \sqrt{N_j(t)}\, e^{i\phi_j(t)}$ and $j = L, R$.

In the last years, we have adopted this mean-field approach to study several problems:

– Josephson junctions with spin-orbit coupling [PRA **89**, 063607 (2014)]
– Josephson junctions assisted by a cavity field [PRA **91**, 023601 (2015)]
– Josephson junctions with atomic losses [PRA **97**, 013602 (2018)]

Mean-field dynamics of the Josephson junction (II)

Introducing the the population imbalance

$$z(t) = \frac{N_L(t) - N_R(t)}{N} \tag{11}$$

and the relative phase

$$\phi(t) = \phi_L(t) - \phi_R(t) \tag{12}$$

one can then derive, from the two-site Bose-Hubbard Hamiltonian, the familiar generalized Josephson equations[1]

$$\frac{d}{dt}z(t) = -\frac{2J}{\hbar}\sqrt{1 - z(t)^2}\sin(\phi(t)) \tag{13}$$

$$\frac{d}{dt}\phi(t) = \frac{NU}{\hbar}z(t) + \frac{2J}{\hbar}\frac{z(t)}{\sqrt{1 - z(t)^2}}\cos(\phi(t)) \tag{14}$$

[1] A. Smerzi, S. Fantoni, S Giovanazzi, S.R. Shenoy, PRL **79**, 4950 (1997).

Mean-field dynamics of the Josephson junction (III)

From a simple linearization of the generalized Josephson equations around $z = 0$ and $\phi = 0$ one gets

$$\frac{d^2}{dt^2} z(t) + \omega_J^2 z(t) = 0 \,, \tag{15}$$

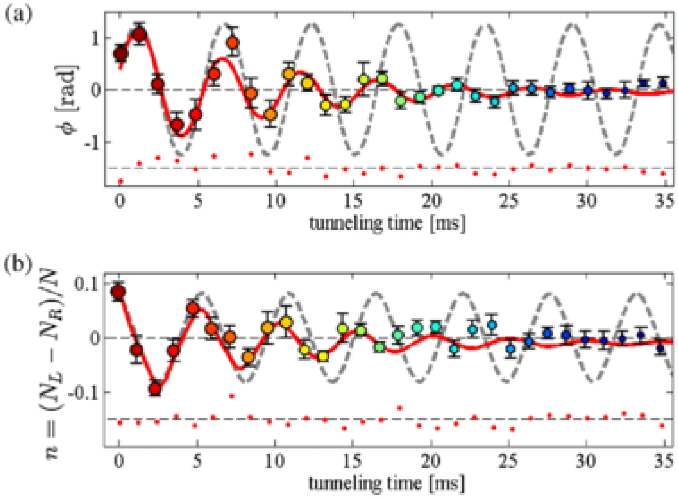
where

$$\omega_J = \frac{2J}{\hbar} \sqrt{1 + \frac{NU}{2J}} \tag{16}$$

is the Josephson frequency of the harmonic oscillation of the population imbalance, and also of the relative phase.
Clearly, for $NU/J \ll 1$ (Rabi regime) the frequency becomes $\omega_J \simeq 2J/\hbar$, while for $NU/J \gg 1$ (Josephson regime) the frequency becomes $\omega_J \simeq \sqrt{2NUJ}/\hbar$.
In any case, assuming a small population imbalance and a small relative phase we have found a pure harmonic dynamics with no dissipation.

Recent experiment: relaxation without dissipation (I)

In a recent experiment with ^{87}Rb atoms [**M. Pigeur et al., PRL 102, 173601 (2018)**] the relaxation of Josephson oscillations in the absence of dissipation has been observed.



Recent experiment: relaxation without dissipation (II)

In this experiment, performed at TU Wien, it has been studied the non-equilibrium tunnel dynamics of $N = 3300$ atoms in the Josephson regime ($UN/J \gg 1$) at ultra-low temperature $T = 18$ nK.
The main results are:

1. Regardless of the initial state and experimental parameters, the dynamics of the relative phase and atom number imbalance shows a relaxation to a phase-locked steady state.
2. Due to the fact that dissipative processes are negligible, the authors of the experiment write that "a microscopic theory compatible with our observations is still missing".
3. The experimental data are not compatible with the mean-field theory based on the generalized Josephson equations. Only including a phenomenological dissipative term in these equations one reproduces the observations.

Exact dynamics of the Josephson junction (I)

The exact quantum dynamics of the bosonic Josephson junction can be obtained assuming that the time-dependent many-body state of the system is given by

$$|\psi(t)\rangle = \sum_{i=0}^{N} c_i(t) \; |i\rangle_L \otimes |N-i\rangle_R \; , \tag{17}$$

where $|i\rangle_L$ is the Fock state of i bosons on the site L and $|N-i\rangle_R$ is the Fock state of $N-i$ bosons on the site R.
We also impose the normalization condition

$$\sum_{i=0}^{N} |c_i(t)|^2 = 1 \; . \tag{18}$$

Exact dynamics of the Josephson junction (II)

The time-dependent Schrödinger equation for the state $|\psi(t)\rangle$ is given by

$$i\hbar \frac{d}{dt}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle \ , \tag{19}$$

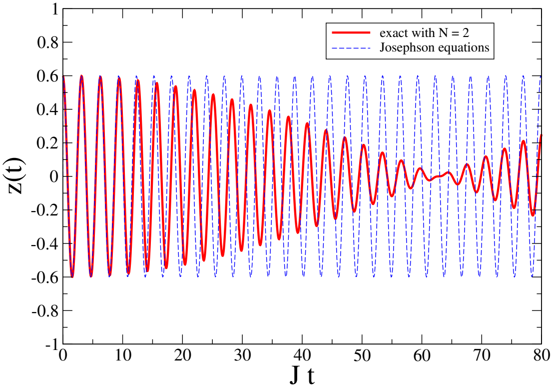
where $\hat{H}$ is the two-site Bose-Hubbard Hamiltonian.
From this equation one finds $N+1$ coupled ordinary differential equations (ODEs) for the time-dependent complex coefficients $c_i(t)$ of the state

$$|\psi(t)\rangle = \sum_{i=0}^{N} c_i(t) \ |i\rangle_L \otimes |N-i\rangle_R \ .$$

For instance, for $N = 2$ one finds these 3 ODEs

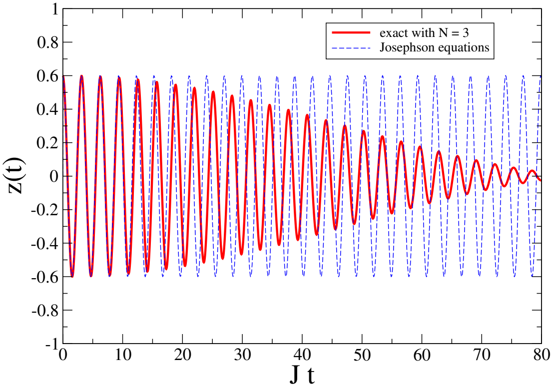
$$i\hbar \frac{d}{dt} \begin{bmatrix} c_0(t) \\ c_1(t) \\ c_2(t) \end{bmatrix} = \begin{bmatrix} U & -\sqrt{2}J & 0 \\ -\sqrt{2}J & 0 & -\sqrt{2}J \\ 0 & -\sqrt{2}J & U \end{bmatrix} \begin{bmatrix} c_0(t) \\ c_1(t) \\ c_2(t) \end{bmatrix} \tag{20}$$

Exact dynamics of the Josephson junction (III)



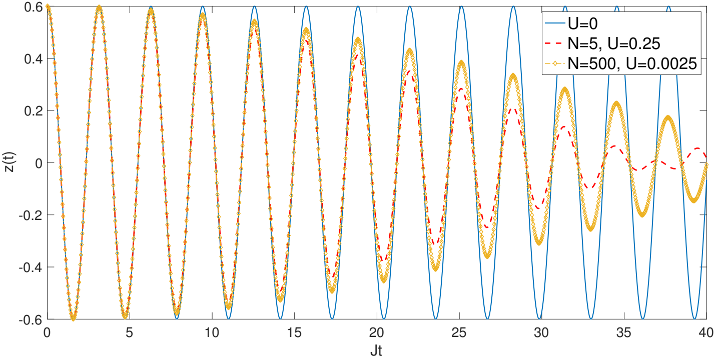
Exact dynamics of the population imbalance for $N = 2$ bosons (solid line) compared with the mean-field (semiclassical) dynamics (dashed line). We choose $J = 10$ and $U = 1/2$ such that $NU = 1$. The initial conditions are $c_0(0) = \sqrt{0.2}$, $c_1(0) = 0$, $c_2(0) = \sqrt{0.8}$.
[S. Wimberger and LS, preliminar results]

Exact dynamics of the Josephson junction (IV)



Exact dynamics of the population imbalance for $N = 3$ bosons (solid line) compared with the mean-field (semiclassical) dynamics (dashed line). We choose $J = 10$ and $U = 1/3$ such that $NU = 1$. The initial conditions are $c_0(0) = \sqrt{0.2}$, $c_1(0) = 0$, $c_2(0) = 0$, $c_3(0) = \sqrt{0.8}$. [S. Wimberger and LS, preliminar results]

Exact dynamics of the Josephson junction (V)



Exact dynamics of the population imbalance for $N = 5$ bosons (red dashed curve) and $N = 500$ bosons (yellow dotted curve) with $J = 10$ and $NU = 1.25$. For comparison there is also the non-interacting case where $U = 0$ (blue solid curve). [S. Wimberger and LS, preliminar results]

Conclusions and open problems

- We have analyzed the ground state $|GS\rangle$ of the two-site Bose-Hubbard Hamiltonian, finding that $|GS\rangle$ is strongly dependent on the ratio U/J: atomic coherent state, twin-Fock state, cat state.
- We have also compared the exact quantum tunneling dynamics with the mean-field one in the Rabi regime ($NU/J \ll 1$), finding that:
  – at small times the mean-field theory is reliable;
  – at large times mean-field dynamics predicts a pure harmonic oscillation while the exact dynamics displays also damping and revival;
  – the damping time T_D slightly increases by increasing the number of bosons N.
- We are **now trying** to compare our exact theoretical results with the recent experimental data obtained at TU Wien in the Josephson regime ($NU/J \gg 1$).

Acknowledgements

Thank you for your attention!