Effects of spin-orbit coupling on the BCS-BEC crossover

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- Conclusions
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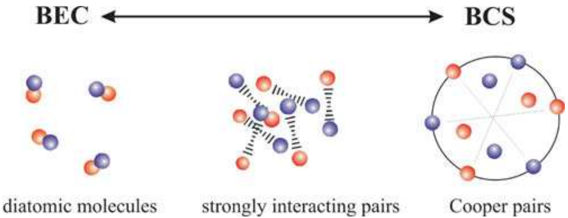
BCS-BEC crossover (I)

In 2004 the BCS-BEC crossover has been observed with **ultracold gases made of fermionic ^{40}K and ^{6}Li alkali-metal atoms.**[1]



This crossover is obtained by changing (with a Feshbach resonance) the s-wave scattering length a_s of the inter-atomic potential:

– $a_s \to 0^-$ (BCS regime of weakly-interacting Cooper pairs)
– $a_s \to \pm\infty$ (unitarity limit of strongly-interacting Cooper pairs)
– $a_s \to 0^+$ (BEC regime of bosonic dimers)

[1] C.A. Regal et al., PRL **92**, 040403 (2004); M.W. Zwierlein et al., PRL **92**, 120403 (2004); M. Bartenstein, A. Altmeyer et al., PRL **92**, 120401 (2004); J. Kinast et al., PRL **92**, 150402 (2004).

BCS-BEC crossover (II)

The crossover from a BCS superfluid ($a_s < 0$) to a BEC of molecular pairs ($a_s > 0$) has been investigated experimentally around a Feshbach resonance, where the s-wave scattering length a_s diverges, and it has been shown that the system is (meta)stable.
The detection of **quantized vortices** under rotation[2] has clarified that **this dilute and ultracold gas of Fermi atoms is superfluid**.
Usually the BCS-BEC crossover is analyzed in terms of

$$y = \frac{1}{k_F a_s} \tag{1}$$

the inverse scaled interaction strength, where $k_F = (3\pi^2 n)^{1/3}$ is the Fermi wave number and n the total density.
The system is dilute because $r_e k_F \ll 1$, with r_e the effective range of the inter-atomic potential.

[2]M.W. Zwierlein *et al.*, Science **311**, 492 (2006); M.W. Zwierlein *et al.*, Nature **442**, 54 (2006).

Artificial spin-orbit coupling

In 2011 and 2012 artificial spin-orbit coupling has been imposed on both bosonic[3] and fermionic[4] atomic gases.
The single-particle Hamiltonian $\hat{h}_{sp}$ with both Rashba and Dresselhaus spin-orbit couplings reads

$$\hat{h}_{sp} = \frac{\hat{p}^2}{2m} + v_R \ (\hat{\sigma}_x \hat{p}_y - \hat{\sigma}_y \hat{p}_x) + v_D \ (\hat{\sigma}_x \hat{p}_y + \hat{\sigma}_y \hat{p}_x) \ , \qquad (2)$$

with $\hat{p}^2 = -\hbar^2 \nabla^2$, $\hat{p}_x = -i\hbar \frac{\partial}{\partial x}$, $\hat{p}_y = -i\hbar \frac{\partial}{\partial y}$, v_R and v_D the Rashba and Dresselhaus couping constant, respectively, and

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \qquad \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} .$$

[3] Y.J. Lin, K. Jimenez-Garcia, and I.B. Spielman, Nature **471**, 83 (2011).
[4] P. Wang et al., PRL **109**, 095301 (2012); L.W. Cheuk et al., PRL **109**, 095302 (2012).

Mean-field approach (I)

The partition function $\mathcal{Z}$ of the uniform two-spin-component Fermi system at temperature T, in a volume V, and with chemical potential μ can be written in terms of a functional integral as

$$\mathcal{Z} = \int \mathcal{D}[\psi_s, \bar{\psi}_s] \; \exp\left\{ -\frac{1}{\hbar} \, S \right\} , \tag{3}$$

where

$$S = \int_0^{\hbar\beta} d\tau \int_V d^3\mathbf{r} \; \mathcal{L} \tag{4}$$

is the Eucidean action functional and $\mathcal{L}$ is the Euclidean Lagrangian density, given by

$$\mathcal{L} = (\bar{\psi}_\uparrow \; , \; \bar{\psi}_\downarrow) \left[\hbar\partial_\tau + \hat{h}_{sp} - \mu \right] \begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix} + g \, \bar{\psi}_\uparrow \, \bar{\psi}_\downarrow \, \psi_\downarrow \, \psi_\uparrow \tag{5}$$

with g is the strength of the s-wave coupling ($g < 0$ in the BCS regime). Notice that $\beta = 1/(k_B T)$ with k_B the Boltzmann constant. In the rest of the seminar we shall use units such that $\hbar = m = k_B = 1$.

Mean-field approach (II)

The Lagrangian density $\mathcal{L}$ is quartic in the fermionic fields ψ_s, but one can reduce the problem to a quadratic Lagrangian density by introducing an auxiliary complex scalar field $\Delta(\mathbf{r}, \tau)$ via Hubbard-Stratonovich transformation[5], which gives

$$\mathcal{Z} = \int \mathcal{D}[\psi_s, \bar{\psi}_s] \, \mathcal{D}[\Delta, \bar{\Delta}] \; \exp\left\{-S_e\right\} , \tag{6}$$

where

$$S_e = \int_0^{1/T} d\tau \int_V d^3\mathbf{r} \; \mathcal{L}_e \tag{7}$$

and the (exact) effective Eucidean Lagrangian density $\mathcal{L}_e$ reads

$$\mathcal{L}_e = (\bar{\psi}_\uparrow \; , \; \bar{\psi}_\downarrow) \left[\partial_\tau + \hat{h}_{sp} - \mu\right] \begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix} + \bar{\Delta} \, \psi_\downarrow \, \psi_\uparrow + \Delta \bar{\psi}_\uparrow \, \bar{\psi}_\downarrow - \frac{|\Delta|^2}{g} \, . \tag{8}$$

[5] H.T.C. Stoof, K.B. Gubbels, D.B.M. Dickerscheid, Ultracold Quantum Fields (Springer, Dordrecht, 2009).

Mean-field approach (III)

It is a standard procedure to integrate out the quadratic fermionic fields and to get a new formally-exact effective action S_{eff} which depends only on the auxiliary field $\Delta(\mathbf{r}, \tau)$. In this way we obtain

$$\mathcal{Z} = \int \mathcal{D}[\Delta, \bar{\Delta}] \quad \exp\left\{-S_{eff}\right\} , \tag{9}$$

where

$$S_{eff} = -Tr[\ln\left(G^{-1}\right)] - \int_0^{1/T} d\tau \int_V d^3\mathbf{r} \, \frac{|\Delta|^2}{g} \tag{10}$$

with $\gamma(\hat{\mathbf{p}}) = v_R(\hat{p}_y + i\hat{p}_x) + v_D(\hat{p}_y - i\hat{p}_x)$ and

$$G^{-1} = \begin{pmatrix} \partial_\tau + \frac{\hat{p}^2}{2m} - \mu & \Delta & \gamma(\hat{\mathbf{p}}) & 0 \\ \bar{\Delta} & \partial_\tau - \frac{\hat{p}^2}{2m} + \mu & 0 & -\gamma(-\hat{\mathbf{p}}) \\ \bar{\gamma}(\hat{\mathbf{p}}) & 0 & \partial_\tau + \frac{\hat{p}^2}{2m} - \mu & \Delta \\ 0 & -\bar{\gamma}(-\hat{\mathbf{p}}) & \bar{\Delta} & \partial_\tau - \frac{\hat{p}^2}{2m} + \mu \end{pmatrix} \tag{11}$$

Mean-field approach (IV)

For a uniform Fermi superfluid within the **simplest mean-field approximation** one has a constant and real gap parameter, i.e. $\Delta(\mathbf{r},\tau)=\Delta$, and the partition function becomes[6]

$$\mathcal{Z}_{mf}=\exp\left\{-S_{mf}\right\}=\exp\left\{-\frac{\Omega_{mf}}{T}\right\}, \qquad (12)$$

where

$$S_{mf}=\frac{\Omega_{mf}}{T}=-\sum_{\mathbf{k}}\left[\sum_{j=1}^{4}\ln\left(1+e^{-E_{\mathbf{k},j}/T}\right)-\frac{\xi_k}{T}\right]-\frac{V}{T}\frac{\Delta^2}{g} \qquad (13)$$

with $\xi_k=\hbar^2k^2/(2m)-\mu$, $\gamma_{\mathbf{k}}=\hbar v_R(k_y+ik_x)+\hbar v_D(k_y-ik_x)$, and

$$E_{\mathbf{k},1} = \sqrt{(\xi_k-|\gamma_{\mathbf{k}}|)^2+\Delta^2}, \qquad E_{\mathbf{k},3}=-E_{\mathbf{k},1}, \qquad (14)$$

$$E_{\mathbf{k},2} = \sqrt{(\xi_k+|\gamma_{\mathbf{k}}|)^2+\Delta^2}, \qquad E_{\mathbf{k},4}=-E_{\mathbf{k},2}. \qquad (15)$$

[6] L. Dell'Anna, G. Mazzarella, L.S., PRA **84**, 033633 (2011).

Mean-field approach (V)

The constant and real gap parameter Δ is obtained from

$$\frac{\partial S_{mf}}{\partial \Delta} = 0 \,, \tag{16}$$

which gives the **gap equation**

$$-\frac{1}{g} = \frac{1}{V} \sum_{\mathbf{k}} \sum_{j=1,2} \frac{\tanh\left(E_{\mathbf{k},j}/2T\right)}{4E_{\mathbf{k},j}} \,. \tag{17}$$

The integral on the right side of this equation is formally divergent. However, expressing the bare interaction strength g in terms of the physical scattering length a_s with the formula[7]

$$-\frac{1}{g} = -\frac{1}{4\pi a_s} + \frac{1}{V} \sum_{\mathbf{k}} \frac{1}{k^2} \tag{18}$$

one obtains the **regularized gap equation**[8]

$$-\frac{1}{4\pi a_s} = \frac{1}{V} \sum_{\mathbf{k}} \left[\sum_{j=1,2} \frac{\tanh\left(E_{\mathbf{k},j}/2T\right)}{4E_{\mathbf{k},j}} - \frac{1}{k^2} \right] \,. \tag{19}$$

[7] M. Marini, F. Pistolesi, G.C. Strinati, EPJ B **1**, 151 (1998).

[8] L. Dell'Anna, G. Mazzarella, L.S., PRA **84**, 033633 (2011).

Mean-field approach (VI)

From the thermodynamic formula

$$N = -\left(\frac{\partial \Omega_{mf}}{\partial \mu}\right)_{V,T} \tag{20}$$

one obtains also the **equation for the number of particles**[9]

$$N = \sum_{\mathbf{k}} \left(1 - \frac{\xi_k - |\gamma_{\mathbf{k}}|}{2E_{\mathbf{k},1}} \tanh\left(E_{\mathbf{k},1}/2T\right) - \frac{\xi_k + |\gamma_{\mathbf{k}}|}{2E_{\mathbf{k},2}} \tanh\left(E_{\mathbf{k},2}/2T\right)\right) . \tag{21}$$

[9] L. Dell'Anna, G. Mazzarella, L.S., PRA **84**, 033633 (2011).

Singlet and triplet condensation (I)

In a Fermi system the largest eigenvalue N_C of the two-body density matrix gives the **number of correlated fermion pairs which have their center of mass with zero linear momentum.**[10] This condensed number of pairs is given by

$$N_C = N_0 + N_1 \ , \tag{22}$$

where

$$N_0 = \int d^3\mathbf{r}\ d^3\mathbf{r}' \left[\ |\langle\, \psi_\downarrow(\mathbf{r})\ \psi_\uparrow(\mathbf{r}')\,\rangle|^2 + |\langle\, \psi_\uparrow(\mathbf{r})\ \psi_\downarrow(\mathbf{r}')\,\rangle|^2 \ \right] \tag{23}$$

is the condensed number of pairs in the spin 0 state ($m_s = 0$), while

$$N_1 = \int d^3\mathbf{r}\ d^3\mathbf{r}' \left[\ |\langle\, \psi_\uparrow(\mathbf{r})\ \psi_\uparrow(\mathbf{r}')\,\rangle|^2 + |\langle\, \psi_\downarrow(\mathbf{r})\ \psi_\downarrow(\mathbf{r}')\,\rangle|^2 \ \right] . \tag{24}$$

is the condensed number of pairs in the spin 1 state ($|m_s| = 1$).

[10] A.J. Leggett, Quantum liquids. Bose condensation and Cooper pairing in condensed-matter systems (Oxford Univ. Press, Oxford, 2006).

Singlet and triplet condensation (II)

In our superfluid Fermi system with spin-orbit coupling we obtain[11]

$$N_0 = \frac{\Delta^2}{4} \sum_{\mathbf{k}} \left(\frac{1}{2E_{\mathbf{k},1}} \tanh\left(E_{\mathbf{k},1}/2T\right) + \frac{1}{2E_{\mathbf{k},2}} \tanh\left(E_{\mathbf{k},2}/2T\right) \right)^2 . \quad (25)$$

and

$$N_1 = \frac{\Delta^2}{4} \sum_{\mathbf{k}} \left(\frac{1}{2E_{\mathbf{k},1}} \tanh\left(E_{\mathbf{k},1}/2T\right) - \frac{1}{2E_{\mathbf{k},2}} \tanh\left(E_{\mathbf{k},2}/2T\right) \right)^2 . \quad (26)$$

Notice that **in the absence of spin-orbit coupling** ($v_R = v_D = 0$) one has $E_{\mathbf{k},1} = E_{\mathbf{k},2}$ from which one gets $N_1 = 0$, and consequently **the condensate number of Cooper pairs in the triplet state is zero**.

[11] L. Dell'Anna, G. Mazzarella, L.S., PRA **84**, 033633 (2011).

Results with Rashba coupling (I)

We are interested in the low temperature regime where the condensate fraction can be quite large. Quantitatively we restrict our study to the **zero temperature limit (T=0)**. In the equations above we have therefore simply $\tanh(E_{\mathbf{k},j}/2T) \to 1$.
In this way the **regularized gap equation** is given by

$$-\frac{1}{4\pi a_s} = \frac{1}{V}\sum_{\mathbf{k}}\left[\sum_{j=1,2}\frac{1}{4E_{\mathbf{k},j}} - \frac{1}{k^2}\right] , \qquad (27)$$

while the **number equation** reads

$$N = \sum_{\mathbf{k}}\left(1 - \frac{\xi_k - |\gamma_{\mathbf{k}}|}{2E_{\mathbf{k},1}} - \frac{\xi_k + |\gamma_{\mathbf{k}}|}{2E_{\mathbf{k},2}}\right) . \qquad (28)$$

Results with Rashba coupling (II)

Similarly, we obtain for the spin 0 condensate number

$$N_0 = \frac{\Delta^2}{4} \sum_{\mathbf{k}} \left(\frac{1}{2E_{\mathbf{k},1}} + \frac{1}{2E_{\mathbf{k},2}} \right)^2 . \tag{29}$$

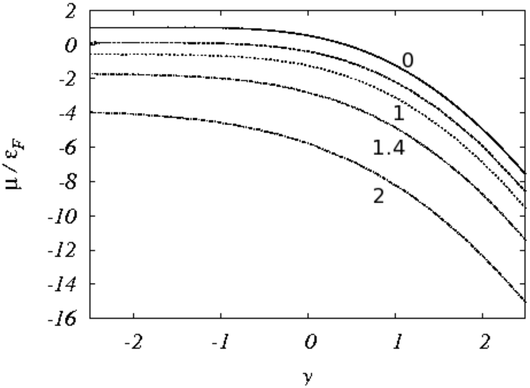
and for the spin 1 condensate number

$$N_1 = \frac{\Delta^2}{4} \sum_{\mathbf{k}} \left(\frac{1}{2E_{\mathbf{k},1}} - \frac{1}{2E_{\mathbf{k},2}} \right)^2 . \tag{30}$$

From the previous equations one can calculate the chemical potential μ, the energy gap Δ, and also the condensate fractions $N_0/(N/2)$ and $N_1/(N/2)$, as a function of the scaled interaction strength $y = 1/(k_F a_S)$.

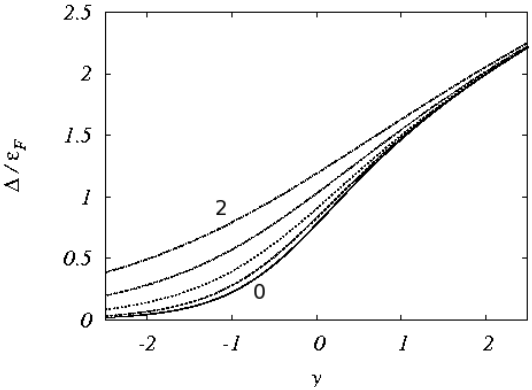
Note: We now show the results obtained for $v_D = 0$, i.e. when **only Rashba spin-orbit coupling is active**.

Results with Rashba coupling (III)



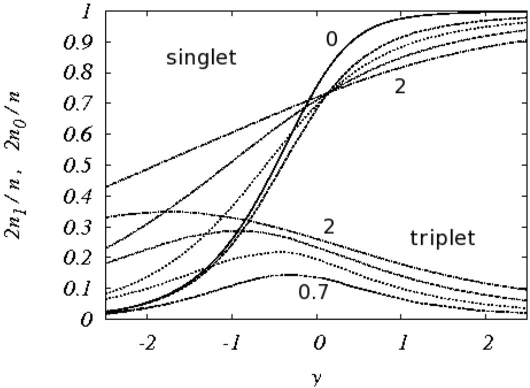
Scaled chemical potential μ/ϵ_F as a function of the adimensional interaction strength $y = 1/(k_F a_s)$ for different values of the scaled Rashba velocity: $v_R/v_F = 0$ (solid line), $v_R/v_F = 0.7$ (long-dashed line), $v_R/v_F = 1$ (short-dashed line), $v_R/v_F = 1.4$ (dotted line), $v_R/v_F = 2$ (dashed-dotted line). Here $\epsilon_F = v_F^2/2$ is the Fermi energy and $v_F = (3\pi^2 n)^{1/3}$ is the Fermi velocity.

Results with Rashba coupling (IV)



Scaled energy gap Δ/ϵ_F as a function of the adimensional interaction strength $y = 1/(k_F a_s)$ for different values of the scaled Rashba velocity: $v_R/v_F = 0$ (solid line), $v_R/v_F = 0.7$ (long-dashed line), $v_R/v_F = 1$ (short-dashed line), $v_R/v_F = 1.4$ (dotted line), $v_R/v_F = 2$ (dashed-dotted line). Here $\epsilon_F = v_F^2/2$ is the Fermi energy and $v_F = (3\pi^2 n)^{1/3}$ is the Fermi velocity.

Results with Rashba coupling (V)



Spin 0 condensate fraction $n_0/(n/2)$ (upper curves) and spin 1 condensate fraction $n_1/(n/2)$ (lower curves) as a function of the adimensional interaction strength $y = 1/(k_F a_s)$ for different values of the scaled Rashba velocity: $v_R/v_F = 0$ (solid line), $v_R/v_F = 0.7$ (long-dashed line), $v_R/v_F = 1$ (short-dashed line), $v_R/v_F = 1.4$ (dotted line), $v_R/v_F = 2$ (dashed-dotted line). Here $v_F = (3\pi^2 n)^{1/3}$ is the Fermi velocity.

Including Dresselhaus coupling (I)

We now consider also the Dresselhaus coupling, i.e. $v_D \neq 0$.
For simplicity we set[12]

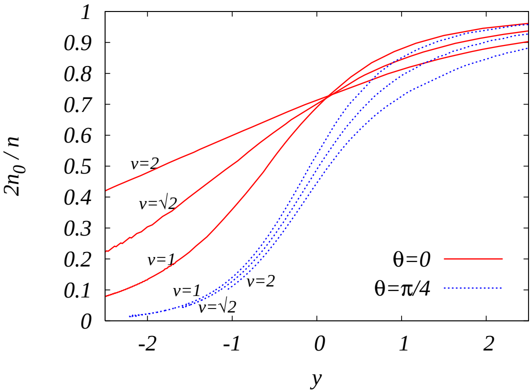
$$v_R = v \cos(\theta) , \quad (31)$$

$$v_D = v \sin(\theta) , \quad (32)$$

where θ is the mixing angle.

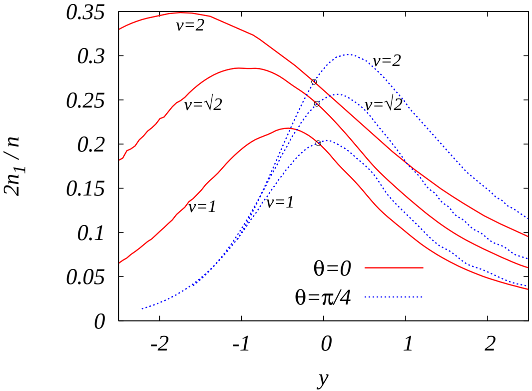
[12] L. Dell'Anna, G. Mazzarella, L.S., PRA **86**, 053632 (2012).

Including Dresselhaus coupling (II)



Spin 0 condensate fraction $n_0/(n/2)$ as a function of the adimensional interaction strength $y = 1/(k_F a_s)$ for three values of v and two values of θ: $\theta = 0$ means $v_R = v$ and $v_D = 0$ (red solid curves), while $\theta = \pi/4$ means $v_R = v_D = (\sqrt{2}/2)v$ (blue dotted curves). v is in units of the Fermi velocity $v_F = (3\pi^2 n)^{1/3}$.

Including Dresselhaus coupling (III)



Spin 1 condensate fraction $n_1/(n/2)$ as a function of the adimensional interaction strength $y = 1/(k_F a_s)$ for three values of v and two values of θ: $\theta = 0$ means $v_R = v$ and $v_D = 0$ (red solid curves), while $\theta = \pi/4$ means $v_R = v_D = (\sqrt{2}/2)v$ (blue dotted curves). v is in units of the Fermi velocity $v_F = (3\pi^2 n)^{1/3}$.

Conclusions

- Unlike the chemical potential μ and the pairing gap Δ which exhibit no particular behavior at the crossover, the condensate fraction is very interesting.
- A finite condensate fraction of spin 1 pairs appears due to the spin-orbit coupling.
- The spin 1 condensate fraction is a not monotonic function of the interaction strength y.

Acknowledgments

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