Broad and Narrow Fano-Feshbach Resonances: Condensate Fraction in the BCS-BEC Crossover

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Summary

- BCS-BEC crossover in 3D and 2D
- Narrow and broad Fano-Feshbach resonances in 3D
- Condensate fraction of Cooper pairs and molecules
- New results for 2D BCS-BEC crossover
- Conclusions
In 2004 the 3D BCS-BEC crossover has been observed with ultracold gases made of two-component fermionic $^{40}$K or $^6$Li alkali-metal atoms.$^1$

This crossover is obtained by using a Fano-Feshbach resonance to change the 3D s-wave scattering length $a_s$ of the inter-atomic potential

$$a_s = a_{bg} \left( 1 + \frac{\Delta_B}{B - B_0} \right) ,$$

where $B$ is the external magnetic field.

$^{1}$C.A. Regal et al., PRL 92, 040403 (2004); M.W. Zwierlein et al., PRL 92, 120403 (2004); J. Kinast et al., PRL 92, 150402 (2004).
The 3D crossover from a BCS superfluid \( (a_s < 0) \) to a BEC of molecular pairs \( (a_s > 0) \) has been investigated experimentally around a Fano-Feshbach resonance, where the 3D s-wave scattering length \( a_s \) diverges, and it has been shown that the system is (meta)stable. The detection of quantized vortices under rotation\(^2\) has clarified that this dilute gas of ultracold atoms is superfluid. Usually the 3D BCS-BEC crossover is analyzed in terms of

\[
y = \frac{1}{k_F a_s}
\]  

(2)

the inverse scaled interaction strength, where \( k_F = \left(3\pi^2 n\right)^{1/3} \) is the Fermi wave number and \( n \) the total density. The system is dilute because \( r_e k_F \ll 1 \), with \( r_e \) the effective range of the inter-atomic potential.

In the last few years also the 2D BEC-BEC crossover has been achieved\textsuperscript{3} with a \textit{quasi-2D Fermi gas of two-component $^6\text{Li}$ atoms}. In the 2D attractive system fermions \textit{always} form biatomic molecules with bound-state energy

$$
\epsilon_B \simeq \frac{\hbar^2}{ma_s^2},
$$

(3)

where $a_s$ is the 2D s-wave scattering length. Both in 3D and 2D the \textit{fermionic single-particle spectrum} is given by

$$
E_{sp}(k) = \sqrt{\left( \frac{\hbar^2 k^2}{2m} - \mu \right)^2 + \Delta^2},
$$

(4)

where $\Delta$ is the \textit{energy gap} and $\mu$ is the \textit{chemical potential}: $\mu > 0$ corresponds to the BCS regime while $\mu < 0$ corresponds to the BEC regime. Moreover, in the deep BEC regime $\mu \rightarrow -\epsilon_B/2$.

\textsuperscript{3}V. Makhalov, K. Martiyanov, and A. Turlapov, PRL 112, 045301 (2014); M.G. Ries et al., PRL 114, 230401 (2015).
The simplest two-channel model\(^4\) of BCS-BEC crossover is given by

\[
\mathcal{L} = \sum_{\sigma=\uparrow, \downarrow} \psi_\sigma^* \left[ \hbar \frac{\partial}{\partial \tau} - \frac{\hbar^2}{2m} \nabla^2 - \mu \right] \psi_\sigma + \phi^* \left[ \hbar \frac{\partial}{\partial \tau} - \frac{\hbar^2}{4m} \nabla^2 - 2\mu + \nu \right] \phi \\
+ g \left( \phi^* \psi_\downarrow \psi_\uparrow + \phi \psi_\uparrow^* \psi_\downarrow^* \right) \tag{5}
\]

where \(\psi_\sigma(\mathbf{r}, \tau)\) is field of fermionic atoms (open channel) and \(\phi(\mathbf{r}, \tau)\) is the field of bosonic biatomic molecules (closed channel).

In the two-channel model $\nu$ is the adjustable threshold energy and $g$ is the Bose-Fermi coupling associated to the Feshbach resonance. In this model the effective atomic coupling strength reads\(^5\)

$$g_{\text{eff}} = \frac{g^2}{\nu - 2\mu} \equiv \frac{4\pi \hbar^2}{m} a_s .$$ (6)

Introducing the adimensional atom-molecule coupling

$$\gamma = g \frac{n^{1/2}}{\epsilon_F} ,$$ (7)

with $n$ the number density and $\epsilon_F$ the Fermi energy, the 3D system is characterized by narrow resonance under the condition $\gamma \ll 1$. Instead there is a broad resonance under the condition $\gamma \gg 1$.

Upper panel: Chemical potential $\mu/\epsilon_F$ as a function of the scaled inverse scattering length $1/(k_F a_s)$.
Lower panel: Effective energy gap $\Delta = g \langle \phi \rangle$ of fermionic excitations. The curves correspond to different values of the scaled atom-molecule coupling $\gamma = gn^{1/2}/\epsilon_F$. [LS, PRA 86, 055602 (2012).]
In the two-channel model the total \textit{condensate density} \( n_0 \) of the system is the sum of two contributions\(^6\), namely

\[
n_0 = n_{F,0} + n_{B,0},
\]

where

\[
n_{F,0} = |\langle \psi_\uparrow \psi_\downarrow \rangle|^2
\]

is the \textit{condensate density} of Cooper-paired atoms and

\[
n_{B,0} = |\langle \phi \rangle|^2
\]

is the \textit{condensate density} of “preformed” bosonic molecules.

\(^6\text{LS, PRA 86, 055602 (2012).}\)
Upper panel: Total condensate fraction \( n_0/n \) as a function of the scaled inverse scattering length \( 1/(k_F a_s) \).
Middle panel: Condensate fraction \( n_{F,0}/n \) of Cooper paired atoms.
Lower panel: Condensate fraction \( n_{B,0}/n \) of atoms in “preformed” molecules.

The curves correspond to different values of the scaled atom-molecule coupling \( \gamma = g n^{1/2}/\epsilon_F \). [LS, PRA 86, 055602 (2012).]
Let us consider the one-channel model of the uniform system at temperature $T$ in two dimensions and with chemical potential $\mu$. The partition function is given by

$$Z = \int \mathcal{D}[\psi_\sigma, \psi_\sigma^*] \exp \left\{ -\frac{1}{\hbar} \int_0^{\hbar \beta} d\tau \int_{L^2} d^2 r \mathcal{L} \right\}, \quad (11)$$

where $\beta \equiv 1/(k_B T)$ with $k_B$ Boltzmann’s constant and

$$\mathcal{L} = \sum_{\sigma = \uparrow, \downarrow} \psi_\sigma^* \left[ \hbar \frac{\partial}{\partial \tau} - \frac{\hbar^2}{2m} \nabla^2 - \mu \right] \psi_\sigma + g \Lambda \psi_\uparrow^* \psi_\downarrow^* \psi_\downarrow \psi_\uparrow \quad (12)$$

is the one-channel Lagrangian density. We are interested in the grand potential $\Omega$, given by

$$\Omega = -\frac{1}{\beta} \ln (Z) \simeq -\frac{1}{\beta} \ln (Z_{mf} Z_g) = \Omega_{mf} + \Omega_B, \quad (13)$$

where $Z_{mf}$ is the mean-field partition function and $Z_g$ is the partition function of Gaussian bosonic pairing fluctuations.
In the gas of paired fermions there are fermionic excitations with energy

\[ E_{sp}(k) = \sqrt{\left( \frac{\hbar^2 k^2}{2m} - \mu \right)^2 + \Delta^2} , \]  

(14)

where \( \Delta \) is the pairing gap, but also bosonic collective excitations which can be approximated (in the BEC regime) by the low-momentum energy

\[ E_{col}(q) = \sqrt{\frac{\hbar^2 q^2}{2m} \left( \lambda \frac{\hbar^2 q^2}{2m} + 2m c_s^2 \right)} , \]  

(15)

where \( \lambda \) is the first correction to phonon dispersion \( E_{col}(q) \approx c_s \hbar q \). At the mean-field level only \( E_{sp}(k) \) contributes and one finds\(^7\)

\[ \Omega_{mf} = -\frac{mL^2}{2\pi\hbar^2} \left( \mu + \frac{1}{2} \epsilon_B \right)^2 , \]  

(16)

where \( \epsilon_B \approx \hbar^2/(ma_s^2) \) is the binding energy of two fermions.

\(^7\)M. Randeria, J-M. Duan, and L-Y. Shieh, PRL 62, 981 (1989).
New results for 2D BCS-BEC crossover (III)

In the deep BEC regime of the 2D BCS-BEC crossover, where the chemical potential $\mu$ becomes negative, performing regularization of zero-point fluctuations we have recently found\textsuperscript{8} that the zero-temperature grand potential (including bosonic excitations) is

$$\Omega = -\frac{mL^2}{64\pi \hbar^2} (\mu + \frac{1}{2} \epsilon_B)^2 \ln \left( \frac{\epsilon_B}{2(\mu + \frac{1}{2} \epsilon_B)} \right).$$

(17)

This is exactly Popov's equation of state of 2D Bose gas with chemical potential $\mu_B = 2(\mu + \epsilon_B/2)$ and mass $m_B = 2m$. In this way we have identified the two-dimensional scattering length $a_B$ of composite bosons as

$$a_B = \frac{1}{2^{1/2} e^{1/4}} a_s.$$  

(18)

The value $a_B/a_s = 1/(2^{1/2} e^{1/4}) \simeq 0.551$ is in full agreement with $a_B/a_s = 0.55(4)$ obtained by Monte Carlo calculations\textsuperscript{9}.

\textsuperscript{8}LS and F. Toigo, PRA 91, 011604(R) (2015).
New results for 2D BCS-BEC crossover (IV)

Also at finite temperature beyond-mean-field effects play a relevant role.

Theoretical predictions for the Berezinskii-Kosterlitz-Thouless critical temperature $T_{BKT}$ compared to recent experimental observation\textsuperscript{10} (circles with error bars). [G. Bighin and LS, arXiv:1507.07542v2].

\textsuperscript{10}P.A. Murthy et al., PRL 115, 010401 (2015).
Conclusions

- The two-channel model of 3D BCS-BEC crossover shows measurable differences (energy gap, condensate fraction) between narrow and broad Fano-Fersbach resonances.
- Within the one-channel model, the regularization of zero-point energy gives remarkable analytical results for composite bosons in 2D at zero temperature.
- Also at finite temperature beyond-mean-field effects become relevant in the strong-coupling regime of 2D BCS-BEC crossover ($T_{BKT}$, second sound).
Thank you for your attention!

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