Broad and Narrow Fano-Feshbach Resonances: Condensate Fraction in the BCS-BEC Crossover

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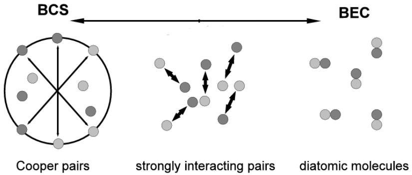
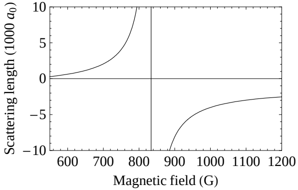
Summary

- BCS-BEC crossover in 3D and 2D
- Narrow and broad Fano-Feshbach resonances in 3D
- Condensate fraction of Cooper pairs and molecules
- New results for 2D BCS-BEC crossover
- Conclusions

BCS-BEC crossover in 3D and 2D (I)

In 2004 the 3D BCS-BEC crossover has been observed with **ultracold gases made of two-component fermionic ^{40}K or ^{6}Li alkali-metal atoms.**[1]





This crossover is obtained by using a Fano-Feshbach resonance to change the 3D s-wave scattering length a_s of the inter-atomic potential

$$a_s = a_{bg} \left(1 + \frac{\Delta_B}{B - B_0} \right) , \qquad (1)$$

where B is the external magnetic field.

[1] C.A. Regal et al., PRL **92**, 040403 (2004); M.W. Zwierlein et al., PRL **92**, 120403 (2004); J. Kinast et al., PRL **92**, 150402 (2004).

BCS-BEC crossover in 3D and 2D (II)

The 3D crossover from a BCS superfluid ($a_s < 0$) to a BEC of molecular pairs ($a_s > 0$) has been investigated experimentally around a Fano-Feshbach resonance, where the 3D s-wave scattering length a_s diverges, and it has been shown that the system is (meta)stable.
The detection of quantized vortices under rotation[2] has clarified that this dilute gas of ultracold atoms is superfluid.
Usually the 3D BCS-BEC crossover is analyzed in terms of

$$y = \frac{1}{k_F a_s} \qquad (2)$$

the inverse scaled interaction strength, where $k_F = (3\pi^2 n)^{1/3}$ is the Fermi wave number and n the total density.
The system is dilute because $r_e k_F \ll 1$, with r_e the effective range of the inter-atomic potential.

[2] M. W. Zwierlein *et al.*, Nature **435**, 1047 (2005); M.W. Zwierlein *et al.*, Science **311**, 492 (2006).

BCS-BEC crossover in 3D and 2D (III)

In the last few years also the 2D BEC-BEC crossover has been achieved[3] with a **quasi-2D Fermi gas of two-component ^{6}Li atoms**. In the 2D attractive system fermions always form biatomic molecules with bound-state energy

$$\epsilon_B \simeq \frac{\hbar^2}{m{a_s}^2} \; , \tag{3}$$

where a_s is the 2D s-wave scattering length.
Both in 3D and 2D the fermionic single-particle spectrum is given by

$$E_{sp}(k) = \sqrt{\left(\frac{\hbar^2 k^2}{2m} - \mu\right)^2 + \Delta^2} \; , \tag{4}$$

where Δ is the energy gap and μ is the chemical potential: $\mu > 0$ corresponds to the BCS regime while $\mu < 0$ corresponds to the BEC regime. Moroever, in the deep BEC regime $\mu \to -\epsilon_B/2$.

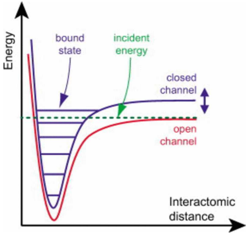
[3] V. Makhalov, K. Martiyanov, and A. Turlapov, PRL **112**, 045301 (2014); M.G. Ries et al., PRL **114**, 230401 (2015).

Narrow and broad Fano-Feshbach resonances in 3D (I)

The simplest two-channel model[4] of BCS-BEC crossover is given by

$$\mathscr{L} = \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^* \left[\hbar\frac{\partial}{\partial\tau} - \frac{\hbar^2}{2m}\nabla^2 - \mu\right]\psi_\sigma + \phi^* \left[\hbar\frac{\partial}{\partial\tau} - \frac{\hbar^2}{4m}\nabla^2 - 2\mu + \nu\right]\phi$$
$$+ \; g\left(\phi^*\,\psi_\downarrow\,\psi_\uparrow + \phi\,\psi_\uparrow^*\,\psi_\downarrow^*\right) \qquad (5)$$

where $\psi_\sigma(\mathbf{r},\tau)$ is field of fermionic atoms (open channel) and $\phi(\mathbf{r},\tau)$ is the field of bosonic biatomic molecules (closed channel).



[4] M. Holland *et al.*, PRL **87**, 120406 (2001); V. Gurarie and L. Raszihovsky, Ann. Phys. **322**, 2 (2007).

Narrow and broad Fano-Feshbach resonances in 3D (II)

In the two-channel model ν is the adjustable threshold energy and g is the Bose-Fermi coupling associated to the Feshbach resonance.
In this model the effective atomic coupling strength reads[5]

$$g_{eff} = \frac{g^2}{\nu - 2\mu} \equiv \frac{4\pi\hbar^2}{m} a_s \ . \tag{6}$$

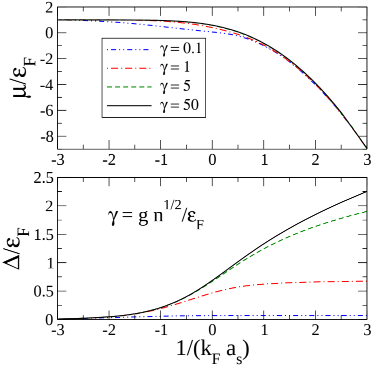
Introducing the adimensional atom-molecule coupling

$$\gamma = g \frac{n^{1/2}}{\epsilon_F} \ , \tag{7}$$

with n the number density and ϵ_F the Fermi energy, the 3D system is characterized by narrow resonance under the condition $\gamma \ll 1$. Instead there is a broad resonance under the condition $\gamma \gg 1$.

[5] Y. Ohashi and Griffin, PRA **72**, 023601 (2005); D.-S. Lee, C.-Y. Lin, R.J. Rivers, PRL **98**, 020603 (2007).

Narrow and broad Fano-Feshbach resonances in 3D (III)



Upper panel: Chemical potential μ/ϵ_F as a function of the scaled inverse scattering length $1/(k_F a_s)$.
Lower panel: Effective energy gap $\Delta = g\langle\phi\rangle$ of fermionic excitations.
The curves correspond to different values of the scaled atom-molecule coupling $\gamma = gn^{1/2}/\epsilon_F$. [LS, PRA **86**, 055602 (2012).]

Condensate fraction of Cooper pairs and molecules (I)

In the two-channel model the total condensate density n_0 of the system is the sum of two contributions[6], namely

$$n_0 = n_{F,0} + n_{B,0} \ , \tag{8}$$

where

$$n_{F,0} = |\langle \psi_\uparrow \psi_\downarrow \rangle|^2 \tag{9}$$

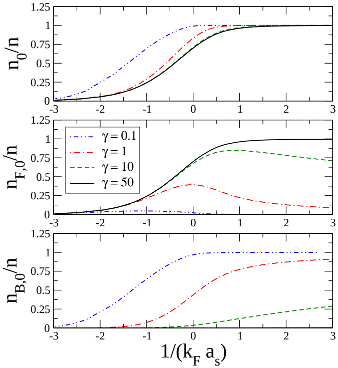
is the condensate density of Cooper-paired atoms and

$$n_{B,0} = |\langle \phi \rangle|^2 \tag{10}$$

is the condensate density of "preformed" bosonic molecules.

[6] LS, PRA **86**, 055602 (2012).

Condensate fraction of Cooper pairs and molecules (II)



Upper panel: Total condensate fraction n_0/n as a function of the scaled inverse scattering length $1/(k_F a_s)$.
Middle panel: Condensate fraction $n_{F,0}/n$ of Cooper paired atoms.
Lower panel: Condensate fraction $n_{B,0}/n$ of atoms in "preformed" molecules.
The curves correspond to different values of the scaled atom-molecule coupling $\gamma = gn^{1/2}/\epsilon_F$. [LS, PRA **86**, 055602 (2012).]

New results for 2D BCS-BEC crossover (I)

Let us consider the one-channel model of the uniform system at temperature T in **two dimensions** and with chemical potential μ. The partition function is given by

$$\mathcal{Z} = \int \mathcal{D}[\psi_\sigma, \psi_\sigma^*] \ \exp\left\{ -\frac{1}{\hbar} \int_0^{\hbar\beta} d\tau \int_{L^2} d^2\mathbf{r} \ \mathscr{L} \right\} , \qquad (11)$$

where $\beta \equiv 1/(k_B T)$ with k_B Boltzmann's constant and

$$\mathscr{L} = \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^* \left[\hbar \frac{\partial}{\partial\tau} - \frac{\hbar^2}{2m} \nabla^2 - \mu \right] \psi_\sigma + g_\Lambda \psi_\uparrow^* \psi_\downarrow^* \psi_\downarrow \psi_\uparrow \qquad (12)$$

is the one-channel Lagrangian density. We are interested in the grand potential Ω, given by

$$\Omega = -\frac{1}{\beta} \ln(\mathcal{Z}) \simeq -\frac{1}{\beta} \ln(\mathcal{Z}_{mf} \mathcal{Z}_g) = \Omega_{mf} + \Omega_B \ , \qquad (13)$$

where $\mathcal{Z}_{mf}$ is the mean-field partition function and $\mathcal{Z}_g$ is the partition function of Gaussian bosonic pairing fluctuations.

New results for 2D BCS-BEC crossover (II)

In the gas of paired fermions there are fermionic excitations with energy

$$E_{sp}(k) = \sqrt{\left(\frac{\hbar^2 k^2}{2m} - \mu\right)^2 + \Delta^2} \,, \tag{14}$$

where Δ is the pairing gap, but also bosonic collective excitations which can be approximated (in the BEC regime) by the low-momentum energy

$$E_{col}(q) = \sqrt{\frac{\hbar^2 q^2}{2m}\left(\lambda \, \frac{\hbar^2 q^2}{2m} + 2 \; m \; c_s^2\right)} \,, \tag{15}$$

where λ is the first correction to phonon dispersion $E_{col}(q) \simeq c_s \hbar q$. At the mean-field level only $E_{sp}(k)$ contributes and one finds[7]

$$\Omega_{mf} = -\frac{mL^2}{2\pi\hbar^2}(\mu + \frac{1}{2}\epsilon_B)^2 \,, \tag{16}$$

where $\epsilon_B \simeq \hbar^2/(m{a_s}^2)$ is the binding energy of two fermions.

[7] M. Randeria, J-M. Duan, and L-Y. Shieh, PRL **62**, 981 (1989).

New results for 2D BCS-BEC crossover (III)

In the deep BEC regime of the **2D BCS-BEC crossover**, where the chemical potential μ becomes negative, performing regularization of zero-point fluctuations we have recently found[8] that the zero-temperature grand potential (including bosonic excitations) is

$$\Omega = -\frac{mL^2}{64\pi\hbar^2}(\mu + \frac{1}{2}\epsilon_B)^2 \ \ln\left(\frac{\epsilon_B}{2(\mu + \frac{1}{2}\epsilon_B)}\right) . \qquad (17)$$

This is exactly Popov's equation of state of 2D Bose gas with chemical potential $\mu_B = 2(\mu + \epsilon_B/2)$ and mass $m_B = 2m$. In this way we have identified the two-dimensional scattering length a_B of composite bosons as

$$\boxed{a_B = \frac{1}{2^{1/2}e^{1/4}} \ a_s \ .} \qquad (18)$$

The value $a_B/a_s = 1/(2^{1/2}e^{1/4}) \simeq 0.551$ is in full agreement with $a_B/a_s = 0.55(4)$ obtained by Monte Carlo calculations[9].

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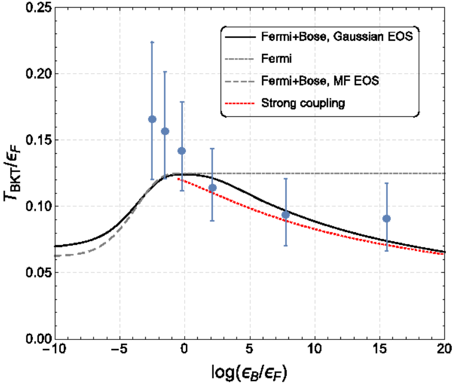
[8] LS and F. Toigo, PRA **91**, 011604(R) (2015).

[9] G. Bertaina and S. Giorgini, PRL **106**, 110403 (2011).

New results for 2D BCS-BEC crossover (IV)

Also at finite temperature beyond-mean-field effects play a relevant role.



Theoretical predictions for the Berezinskii-Kosterlitz-Thouless critical temperature T_{BKT} compared to recent experimental observation[10] (circles with error bars). [G. Bighin and LS, arXiv:1507.07542v2].

[10] P.A. Murthy et al., PRL **115**, 010401 (2015).

Conclusions

- The two-channel model of 3D BCS-BEC crossover shows measurable differences (energy gap, condensate fraction) between narrow and broad Fano-Fershbach resonances.
- Within the one-channel model, the regularization of zero-point energy gives remarkable analytical results for composite bosons in 2D at zero temperature.
- Also at finite temperature beyond-mean-field effects become relevant in the strong-coupling regime of 2D BCS-BEC crossover (T_{BKT}, second sound).

Acknowledgements

Thank you for your attention!

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