Two-dimensional ultracold atomic gases: Kosterlitz-Thouless and collisionless dynamics

Luca Salasnich

Dipartimento di Fisica e Astronomia "Galileo Galilei", Università di Padova
INO-CNR, Research Unit of Sesto Fiorentino, Consiglio Nazionale delle Ricerche

International Conference on Quantum and Atom Optics,
IIT Patna, December 16, 2018

Talk based on the papers
G. Bighin and LS, Phys. Rev. B **93**, 014519 (2016)
A. Cappellaro, F. Toigo, and LS, Phys. Rev. A **98**, 043605 (2018).

Summary

- 2D superfluids and BKT
- Recent experiment at ENS Paris
- 2D Boltzmann-Landau-Vlasov equation
- Collisionless sound and comparison with experiment
- Conclusions

2D superfluids and BKT (I)

BEC phase transition: For an ideal gas of non-interacting identical bosons there is Bose-Einstein condensation (BEC) only below a critical temperature T_{BEC}. In particular one finds

$$k_B\,T_{BEC} = \begin{cases} \frac{1}{2\pi\zeta(3/2)^{2/3}}\frac{\hbar^2}{m}n^{2/3} & \text{for } D=3 \\ 0 & \text{for } D=2 \\ \text{no solution} & \text{for } D=1 \end{cases}$$

where D is the spatial dimension of the system, n is the number density, and $\zeta(x)$ is the Riemann zeta function.

This result due to Einstein (1925), which says that there is no BEC at finite temperature for $D \leq 2$ in the case of non-interacting bosons, was extended to interacting systems by **Mermin** and **Wagner** in 1966.

2D superfluids and BKT (II)

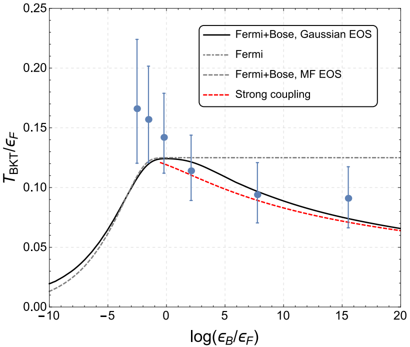
For the BEC phase transition, the Mermin-Wagner theorem[1] states that there is no Bose-Einstein condensation at finite temperature in homogeneous systems with sufficiently short-range interactions in dimensions $D \leq 2$.

Despite the absence of BEC, **Kosterlitz** and **Thouless**[2] (but also **Berezinskii**[3]) suggested that a 2D fluid can be superfluid below a critical temperature, the so-called Berezinskii-Kosterlitz-Thouless (BKT) critical temperature T_c.

The Kosterlitz-Thouless (KT) transition has been observed experimentally in various physical systems: thin films of superfluid ^{4}He (1978), thin films of superconductors (1981), quasi-2D bosonic gas of ^{87}Rb atoms (2006). The KT transition has been observed also in quasi-2D superfluid fermionic gases made of ^{6}Li atoms in the BCS-BEC crossover (2015).

[1] N. D. Mermin and H. Wagner, Phys. Rev. Lett. **17**, 133 (1966).
[2] J. M. Kosterlitz and D. J. Thouless, J. Phys. C: Solid State Phys. **6**, 1181 (1973).
[3] V. L. Berezinskii, Sov. Phys. JETP **34**, 610 (1972).

2D superfluids and BKT (III)



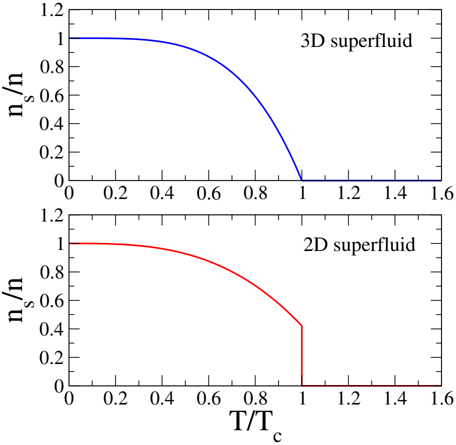
Critical temperature T_{BKT} vs binding energy ϵ_B for a 2D gas of two-component ^{6}Li fermionic atoms. **Circles with error bar** are experimental data.[4] The **solid black curve** is our beyond-mean-field (Gaussian) theory[5] in the 2D BCS-BEC crossover. $\epsilon_F = \hbar^2 \pi n/m$ is the 2D Fermi energy with n the 2D number density.

[4] P. A. Murthy et al., Phys. Rev. Lett. 115, 010401 (2015).

[5] G. Bighin and LS, Phys. Rev. B **93**, 014519 (2016).

2D superfluids and BKT (IV)

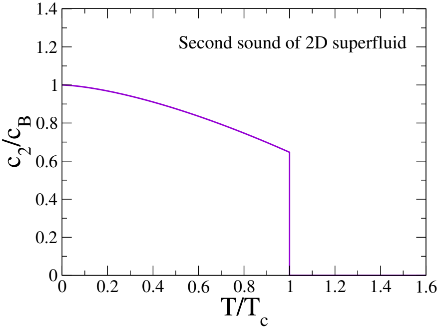
An important prediction of the Kosterlitz-Thouless transition is that, contrary to the 3D case, the superfluid density $n_s(T)$



in 2D jumps to zero at the BKT critical temperature T_c.

2D superfluids and BKT (V)

According to the two-fluid hydrodynamics of Landau-Khalatnikov[6] the **sound velocity** has two branches: first sound velocity c_1 and second sound velocity c_2.

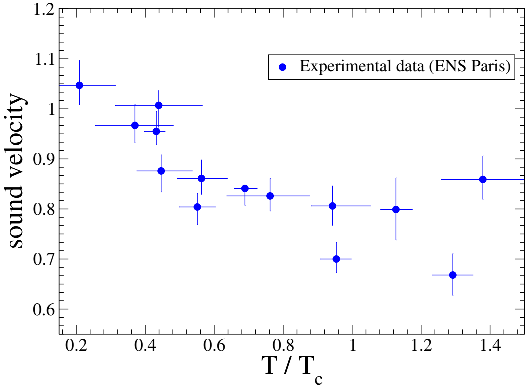


The slower one is the second sound velocity, which also jumps to zero at the BKT critical temperature T_c for 2D bosonic gases.[7]

[6] L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics 10: Physical Kinetics* (Pergamon International Library, Exeter, 1981).

[7] M.Ota and S. Stringari, Phys. Rev. A **97**, 033604 (2018).

Recent experiment at ENS Paris (I)



Recently the speed of sound in a uniform quasi-2D Bose gas of ^{87}Rb atoms has been measured at Ecole Normale Superieure (ENS) of Paris[8].

These experimental results are in agreement with the predictions of the two-fluid hydrodynamics of Landau-Khalatnikov **only** well below the BKT critical temperature T_c.

[8] J. L. Ville *et al.*, Phys. Rev. Lett. **121**, 145301 (2018).

Recent experiment at ENS Paris (II)

The strong discrepancy near and above T_c between the experimental data[9] and the two-fluid hydrodynamics of Landau-Khalatnikov has been explained[10] by suggesting that the experimental conditions are such that in this case collisions are not efficient enough to ensure the local thermodynamic equilibrium required by hydrodynamics and therefore the dynamics is collisionless.

Moreover, above the BKT critical temperature T_c the experimental results show the presence of a **strong damping** of the sound mode.

In the next slides we reproduce **all** the experimental results found at ENS Paris by using the collisionless Boltzmann-Landau-Vlasov equation.

[9] J. L. Ville *et al.*, Phys. Rev. Lett. **121**, 145301 (2018).
[10] M. Ota *et al.*, Phys. Rev. Lett. **121**, 145302 (2018).

2D Boltzmann-Landau-Vlasov equation (I)

Due to the presence of a strong harmonic confinement along the z axis, for the collisionless bosonic system one finds[11] that the planar distribution $f(\mathbf{r}, \mathbf{p})$ of atoms in the 4D single-particle phase space $(\mathbf{r}, \mathbf{p}) = (x, y, p_x, p_y)$ satisfies the 2D Boltzmann-Landau-Vlasov equation

$$\left[\frac{\partial}{\partial t} + \frac{\mathbf{p}}{m} \cdot \nabla_{\mathbf{r}} - \nabla_{\mathbf{r}}\left(\mathcal{U}_{\mathrm{mf}}\right) \cdot \nabla_{\mathbf{p}}\right] f(\mathbf{r}, \mathbf{p}, t) = 0 \, , \qquad (1)$$

where

$$\mathcal{U}_{\mathrm{mf}}(\mathbf{r}, t) = g_{2D} \int \frac{d^2\mathbf{p}}{(2\pi\hbar)^2} \, f(\mathbf{r}, \mathbf{p}, t) \qquad (2)$$

is the Hartree-Fock mean-field term and

$$g_{2D} = \frac{\sqrt{8\pi}\hbar^2}{m} \left(\frac{a_s}{a_z}\right) \qquad (3)$$

with a_s the 3D scattering length and a_z the characteristic length of the harmonic confinement along the z axis.

[11] F. Baldovin, A. Cappellaro, E. Orlandini, and LS, J. Stat. Mech. 063303 (2016).

2D Boltzmann-Landau-Vlasov equation (II)

We set

$$f(\mathbf{r}, \mathbf{p}, t) = f_0(\mathbf{p}) + \delta f(\mathbf{r}, \mathbf{p}, t) \tag{4}$$

where $f_0(\mathbf{p})$ is a stationary and isotropic distribution and $\delta f(\mathbf{r}, \mathbf{p}, t)$ a very small perturbation around it.
It follows that the linearized equation for $\delta f(\mathbf{r}, \mathbf{p}, t)$ reads

$$\left[\frac{\partial}{\partial t} + \frac{\mathbf{p}}{m} \cdot \nabla_{\mathbf{r}}\right] \delta f(\mathbf{r}, \mathbf{p}, t) = g_{2D} \int \frac{d^2\mathbf{p}'}{(2\pi\hbar)^2} \nabla_{\mathbf{r}} \delta f(\mathbf{r}, \mathbf{p}', t) \cdot \nabla_{\mathbf{p}} f_0(\mathbf{p}) \, . \tag{5}$$

Performing the Fourier transform of this equation according to $\widehat{\delta f}(\mathbf{k}, \mathbf{p}, \omega) = \int dt \int d^2\mathbf{r} \, \delta f(\mathbf{r}, \mathbf{p}, t) \exp\left(i(\mathbf{k} \cdot \mathbf{r} - \omega t)\right)$ with $\mathbf{k}$ the 2D wavevector and ω the angular frequency, one finds an implicit formula for the dispersion relation, given by

$$1 - g_{2D} \int \frac{d^2\mathbf{p}}{(2\pi\hbar)^2} \, \frac{\mathbf{k} \cdot \nabla_{\mathbf{p}} f_0(\mathbf{p})}{\mathbf{p} \cdot \mathbf{k}/m - \omega} = 0 \, . \tag{6}$$

2D Boltzmann-Landau-Vlasov equation (III)

In Eq. (6) there is a singularity on the integration path for $\omega = \mathbf{p} \cdot \mathbf{k}/m$. In order to attach a meaning to the integral, we must interpret ω as a complex quantity, i.e.

$$\omega = \omega_R + i\, \omega_I \ , \tag{7}$$

where $\omega_I > 0$ in order to avoid an exponential growth of the perturbation. Eq. (6) can be further simplified by assuming, without loss of generality, that $\mathbf{k} = (k, 0)$. In this way one finds

$$1 - g_{2D} \int \frac{dp_x}{(2\pi\hbar)} \frac{\partial \tilde{f}_0(p_x)}{\partial p_x} \frac{1}{\frac{p_x}{m} - c} = 0 \tag{8}$$

where

$$c = \frac{\omega}{k} \tag{9}$$

is the collisionless sound and $\tilde{f}_0(p_x) = \int f_0(p_x, p_y) dp_y/(2\pi\hbar)$.

Collisionless sound and comparison with experiment (I)

We choose the Bose-Einstein distribution function

$$f_0(\mathbf{p}) = \frac{1}{L^2} \frac{1}{e^{(\frac{p^2}{2m}+g_{2D}n-\mu)/(k_B T)} - 1} \simeq \frac{1}{L^2} \left(\frac{k_B T}{\frac{p^2}{2m}} - \tilde{\mu} \right) \tag{10}$$

as the equilibrium distribution of 2D weakly-interacting bosonic atoms with $n = N/L^2$, where k_B is the Boltzmann constant and T is the absolute temperature. The Hartree interaction term $g_{2D}n$ can be formally removed by introducing a shifted chemical potential $\tilde{\mu} = \mu - g_{2D}n$.
From the normalization condition

$$N = \int \frac{d^2\mathbf{r} d^2\mathbf{p}}{(2\pi\hbar)^2} f_0(\mathbf{p}) \ , \tag{11}$$

one finds the equation of state

$$\tilde{\mu} = k_B T \ \ln\left(1 - e^{-T_B/T}\right) \tag{12}$$

where $k_B T_B = 2\pi\hbar^2 n/m$ is the temperature of Bose degeneracy and clearly $\tilde{\mu} < 0$. The approximate expression for $f_0(\mathbf{p})$ is valid under the condition $T \ll T_B$ (degenerate regime).

Collisionless sound and comparison with experiment (II)

Under the further assumption that $c = c_R + i c_I$ with $c_I \ll c_R$, the coupled equations for the real and imaginary part of the zero-sound velocity read

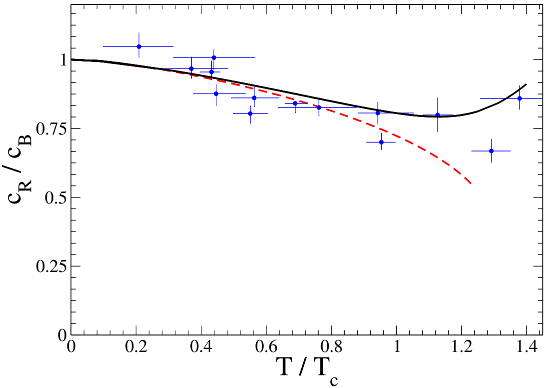
$$
\begin{aligned}
1 \quad &+ \quad \frac{\tilde{g}_{2D} k_B T}{2\pi} \left[\frac{2}{m c_R{}^2 - 2\tilde{\mu}} + \frac{\sqrt{m c_R{}^2}}{(m c_R{}^2 - 2\tilde{\mu})^{3/2}} \ln \left(\frac{\sqrt{m c_R{}^2 - 2\tilde{\mu}} - \sqrt{m c_R{}^2}}{\sqrt{m c_R{}^2 - 2\tilde{\mu}} + \sqrt{m c_R{}^2}} \right) \right] \\
&+ \quad \tilde{g}_{2D} k_B T c_I \frac{m c_R{}^2 + \tilde{\mu}}{\sqrt{m}(m c_R{}^2 - 2\tilde{\mu})^{5/2}} = 0 \,,
\end{aligned}
\tag{13}
$$

$$
c_I = - \frac{\frac{c_R}{\sqrt{m}(m c_R{}^2 - 2\tilde{\mu})^{3/2}}}{\frac{6\, c_R}{(m c_R{}^2 - 2\tilde{\mu})^2} + \frac{2(m c_R{}^2 + \tilde{\mu})}{\sqrt{m}(m c_R{}^2 - 2\tilde{\mu})^{5/2}} \log \left(\frac{\sqrt{m c_R{}^2 - 2\tilde{\mu}} - \sqrt{m c_R{}^2}}{\sqrt{m c_R{}^2 - 2\tilde{\mu}} + \sqrt{m c_R{}^2}} \right)} \,, \tag{14}
$$

where

$$
\tilde{g}_{2D} = \frac{m}{\hbar^2} g_{2D} = \sqrt{8\pi} \frac{a_s}{a_z} \,. \tag{15}
$$

Collisionless sound and comparison with experiment (III)

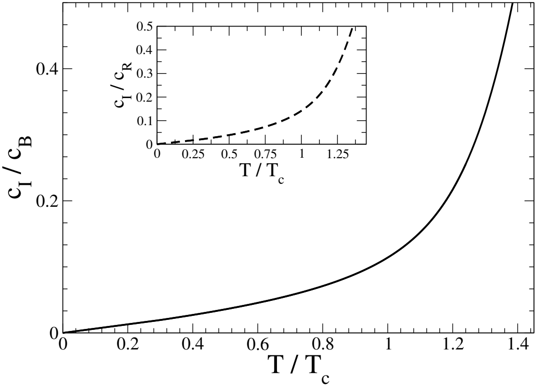


Sound velocity c_R in units of $c_B = \sqrt{g_{2D} n/m}$ vs T/T_c for $\tilde{g}_{2D} = 0.16$. The **solid black line** represents our theoretical prediction[12] while the blue dots are the experimental data obtained at ENS Paris.[13] The red dashed line is obtained by our equations with $c_l = 0$. Here $T_c = 0.13\ T_B$.

[12] A. Cappellaro, F. Toigo, and LS, Phys. Rev. A **98**, 043605 (2018).

[13] J. L. Ville *et al.*, Phys. Rev. Lett. **121**, 145301 (2018).

Collisionless sound and comparison with experiment (IV)



Imaginary part c_I of the sound velocity in units of $c_B = \sqrt{g_{2D} n/m}$ as a function of the scaled temperature T/T_c for $\tilde{g}_{2D} \simeq 0.16$.
Inset: Ratio c_I/c_R between the imaginary and the real part of the sound velocity c as a function of the temperature.

Conclusions

- We have analyzed the sound propagation in collisionless bosonic gases assuming a 2D configuration.
- By solving the linearized 2D Boltzmann-Landau-Vlasov equation in the degenerate regime, we have:
  – derived two coupled algebraic equations for the real and imaginary part of the sound velocity;
  – compared our theoretical results with the recent experiment at ENS Paris;
  – found a very good agreement betweeen our theory and the experiment.
- Our theoretical analysis strongly suggests that the density perturbation used in the experiment has excited the "bosonic zero sound", i.e. the sound of a collisionless bosonic fluid.
- The two-fluid hydrodynamics of Landau-Khalatnikov describes correctly only the collisional regime.

Acknowledgements

Thank you for attention!

Main sponsor: Italian Ministry of Education, University and Research (FFABR Grant).