

Collisionless Dynamics in Two-Dimensional Bosonic Gases

Luca Salasnich

Dipartimento di Fisica e Astronomia "Galileo Galilei", Università di Padova
INO-CNR, Research Unit of Sesto Fiorentino, Consiglio Nazionale delle Ricerche

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Talk based on the paper

A. Cappellaro, F. Toigo, and LS, Phys. Rev. A **98**, 043605 (2018).

Summary

- 2D superfluids and BKT
- Recent experiment at ENS Paris
- 2D Boltzmann-Landau-Vlasov equation
- Collisionless sound and comparison with experiment
- Conclusions

2D superfluids and BKT (I)

BEC phase transition: For an ideal gas of non-interacting identical bosons there is Bose-Einstein condensation (BEC) only below a critical temperature T_{BEC} . In particular one finds

$$k_B T_{BEC} = \begin{cases} \frac{1}{2\pi\zeta(3/2)^{2/3}} \frac{\hbar^2}{m} n^{2/3} & \text{for } D = 3 \\ 0 & \text{for } D = 2 \\ \text{no solution} & \text{for } D = 1 \end{cases}$$

where D is the spatial dimension of the system, n is the number density, and $\zeta(x)$ is the Riemann zeta function.

This result due to Einstein (1925), which says that there is no BEC at finite temperature for $D \leq 2$ in the case of non-interacting bosons, was extended to interacting systems by **Mermin** and **Wagner** in 1966.

2D superfluids and BKT (II)

For the BEC phase transition, the **Mermin-Wagner theorem**¹ states that there is no Bose-Einstein condensation at finite temperature in homogeneous systems with sufficiently short-range interactions in dimensions $D \leq 2$.

Despite the absence of BEC, **Kosterlitz** and **Thouless**² (but also **Berezinskii**³) suggested that a 2D fluid can be superfluid below a critical temperature, the so-called **Berezinskii-Kosterlitz-Thouless (BKT) critical temperature** T_c .

The **Kosterlitz-Thouless (KT) transition** has been **observed experimentally in various physical systems**: thin films of superfluid ^4He (1978), thin films of superconductors (1981), quasi-2D bosonic gas of ^{87}Rb atoms (2006). The KT transition has been observed also in quasi-2D superfluid fermionic gases made of ^6Li atoms **in the BCS-BEC crossover** (2015).

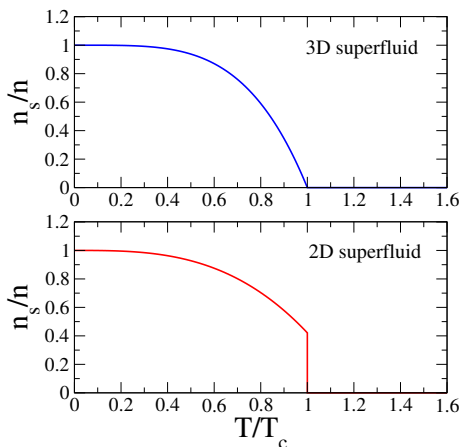
¹N. D. Mermin and H. Wagner, Phys. Rev. Lett. **17**, 133 (1966).

²J. M. Kosterlitz and D. J. Thouless, J. Phys. C: Solid State Phys. **6**, 1181 (1973).

³V. L. Berezinskii, Sov. Phys. JETP **34**, 610 (1972).

2D superfluids and BKT (III)

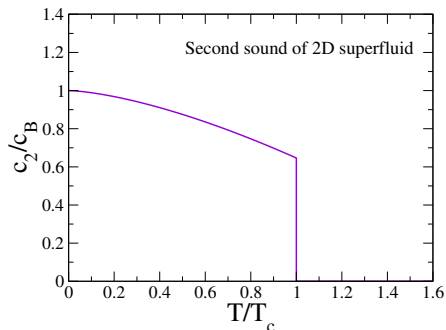
An important prediction of the Kosterlitz-Thouless transition is that, contrary to the 3D case, the **superfluid density** $n_s(T)$



in 2D jumps to zero at the BKT critical temperature T_c .

2D superfluids and BKT (IV)

According to the two-fluid hydrodynamics of Landau-Khalatnikov⁴ the **sound velocity** has two branches: **first sound velocity** c_1 and **second sound velocity** c_2 .

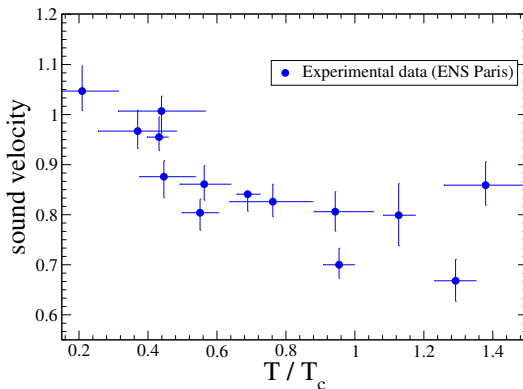


The slower one is the **second sound velocity**, which also **jumps to zero** at the BKT critical temperature T_c for 2D bosonic gases.⁵

⁴L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics 10: Physical Kinetics* (Pergamon International Library, Exeter, 1981).

⁵M.Ota and S. Stringari, *Phys. Rev. A* **97**, 033604 (2018).

Recent experiment at ENS Paris (I)



Recently the speed of sound in a uniform quasi-2D Bose gas of ^{87}Rb atoms has been measured at Ecole Normale Supérieure (ENS) of Paris⁶.

These experimental results are in agreement with the predictions of the two-fluid hydrodynamics of Landau-Khalatnikov **only** well below the BKT critical temperature T_c .

⁶J. L. Ville *et al.*, Phys. Rev. Lett. **121**, 145301 (2018).

Recent experiment at ENS Paris (II)

The strong discrepancy near and above T_c between the **experimental data**⁷ and the two-fluid hydrodynamics of Landau-Khalatnikov has been explained⁸ by suggesting that the experimental conditions are such that in this case **collisions are not efficient enough to ensure the local thermodynamic equilibrium** required by hydrodynamics and therefore **the dynamics is collisionless**.

Moreover, above the BKT critical temperature T_c the **experimental results** show the presence of a **strong damping** of the sound mode.

In the next slides we reproduce **all** the **experimental results** found at ENS Paris by using the **collisionless Boltzmann-Landau-Vlasov equation**.

⁷J. L. Ville *et al.*, Phys. Rev. Lett. **121**, 145301 (2018).

⁸M. Ota *et al.*, Phys. Rev. Lett. **121**, 145302 (2018).

2D Boltzmann-Landau-Vlasov equation (I)

Due to the presence of a strong harmonic confinement along the z axis, for the collisionless bosonic system one finds⁹ that the planar distribution $f(\mathbf{r}, \mathbf{p})$ of atoms in the 4D single-particle phase space

$(\mathbf{r}, \mathbf{p}) = (x, y, p_x, p_y)$ satisfies the **2D Boltzmann-Landau-Vlasov equation**

$$\left[\frac{\partial}{\partial t} + \frac{\mathbf{p}}{m} \cdot \nabla_{\mathbf{r}} - \nabla_{\mathbf{r}}(\mathcal{U}_{\text{mf}}) \cdot \nabla_{\mathbf{p}} \right] f(\mathbf{r}, \mathbf{p}, t) = 0, \quad (1)$$

where

$$\mathcal{U}_{\text{mf}}(\mathbf{r}, t) = g_{2D} \int \frac{d^2\mathbf{p}}{(2\pi\hbar)^2} f(\mathbf{r}, \mathbf{p}, t) \quad (2)$$

is the Hartree-Fock mean-field term and

$$g_{2D} = \frac{\sqrt{8\pi}\hbar^2}{m} \left(\frac{a_s}{a_z} \right) \quad (3)$$

with a_s the **3D scattering length** and a_z the characteristic length of the harmonic confinement along the z axis.

⁹F. Baldovin, A. Cappellaro, E. Orlandini, and LS, J. Stat. Mech. 063303 (2016).

2D Boltzmann-Landau-Vlasov equation (II)

We set

$$f(\mathbf{r}, \mathbf{p}, t) = f_0(\mathbf{p}) + \delta f(\mathbf{r}, \mathbf{p}, t) \quad (4)$$

where $f_0(\mathbf{p})$ is a stationary and isotropic distribution and $\delta f(\mathbf{r}, \mathbf{p}, t)$ a very small perturbation around it.

It follows that the linearized equation for $\delta f(\mathbf{r}, \mathbf{p}, t)$ reads

$$\left[\frac{\partial}{\partial t} + \frac{\mathbf{p}}{m} \cdot \nabla_{\mathbf{r}} \right] \delta f(\mathbf{r}, \mathbf{p}, t) = g_{2D} \int \frac{d^2 \mathbf{p}'}{(2\pi\hbar)^2} \nabla_{\mathbf{r}} \delta f(\mathbf{r}, \mathbf{p}', t) \cdot \nabla_{\mathbf{p}} f_0(\mathbf{p}). \quad (5)$$

Performing the Fourier transform of this equation according to $\widehat{\delta f}(\mathbf{k}, \mathbf{p}, \omega) = \int dt \int d^2 \mathbf{r} \delta f(\mathbf{r}, \mathbf{p}, t) \exp(i(\mathbf{k} \cdot \mathbf{r} - \omega t))$ with \mathbf{k} the 2D wavevector and ω the angular frequency, one finds an implicit formula for the dispersion relation, given by

$$1 - g_{2D} \int \frac{d^2 \mathbf{p}}{(2\pi\hbar)^2} \frac{\mathbf{k} \cdot \nabla_{\mathbf{p}} f_0(\mathbf{p})}{\mathbf{p} \cdot \mathbf{k} / m - \omega} = 0. \quad (6)$$

2D Boltzmann-Landau-Vlasov equation (III)

In Eq. (6) there is a singularity on the integration path for $\omega = \mathbf{p} \cdot \mathbf{k}/m$. In order to attach a meaning to the integral, we must interpret ω as a complex quantity, i.e.

$$\omega = \omega_R + i \omega_I, \quad (7)$$

where $\omega_I > 0$ in order to avoid an exponential growth of the perturbation. Eq. (6) can be further simplified by assuming, without loss of generality, that $\mathbf{k} = (k, 0)$. In this way one finds

$$1 - g_{2D} \int \frac{dp_x}{(2\pi\hbar)} \frac{\partial \tilde{f}_0(p_x)}{\partial p_x} \frac{1}{\frac{p_x}{m} - c} = 0 \quad (8)$$

where

$$c = \frac{\omega}{k} \quad (9)$$

is the **collisionless sound** and $\tilde{f}_0(p_x) = \int f_0(p_x, p_y) dp_y / (2\pi\hbar)$.

Collisionless sound and comparison with experiment (I)

From Eq. (8) one can extract the **speed of sound** c in our collisionless regime. This velocity is, in general, a complex number such that

$$c = \frac{\omega}{k} = c_R + i c_I \quad (10)$$

with $c_R = \omega_R/k$ and $c_I = \omega_I/k$.

In the limit of weakly damped wave, i.e. $c_I \ll c_R$, one finds two coupled equations for c_R and c_I :

$$1 - g_{2D} \mathcal{P} \int \frac{dp_x}{(2\pi\hbar)} \left[\frac{\partial \tilde{f}_0(p_x)/\partial p_x}{p_x/m - c_R} \right] - \pi c_I \frac{\partial \phi(c)}{\partial c} \Big|_{c_R} = 0, \quad (11)$$

$$c_I = \frac{\pi \frac{\partial \tilde{f}_0(p_x)}{\partial p_x} \Big|_{p_x = m c_R}}{\frac{\partial}{\partial c_R} \left\{ \mathcal{P} \int \frac{dp_x}{(2\pi\hbar)} \left[\frac{\partial \tilde{f}_0(p_x)/\partial p_x}{p_x/m - c_R} \right] \right\}}, \quad (12)$$

where we denote $\phi(c) = \frac{m g_{2D}}{(2\pi\hbar)} \frac{\partial \tilde{f}_0}{\partial p_x} \Big|_{p_x = mc}$ and \mathcal{P} means principal value.

Collisionless sound and comparison with experiment (II)

We choose the Bose-Einstein distribution function

$$f_0(\mathbf{p}) = \frac{1}{L^2} \frac{1}{e^{(\frac{p^2}{2m} + g_{2D}n - \mu)/(k_B T)} - 1} \simeq \frac{1}{L^2} \left(\frac{k_B T}{\frac{p^2}{2m}} - \tilde{\mu} \right) \quad (13)$$

as the equilibrium distribution of 2D weakly-interacting bosonic atoms with $n = N/L^2$, where k_B is the Boltzmann constant and T is the absolute temperature. The Hartree interaction term $g_{2D}n$ can be formally removed by introducing a shifted chemical potential $\tilde{\mu} = \mu - g_{2D}n$. From the normalization condition

$$N = \int \frac{d^2\mathbf{r}d^2\mathbf{p}}{(2\pi\hbar)^2} f_0(\mathbf{p}), \quad (14)$$

one finds the equation of state

$$\tilde{\mu} = k_B T \ln \left(1 - e^{-T_B/T} \right) \quad (15)$$

where $k_B T_B = 2\pi\hbar^2 n/m$ is the temperature of Bose degeneracy and clearly $\tilde{\mu} < 0$. The approximate expression for $f_0(\mathbf{p})$ is valid under the condition $T \ll T_B$ (degenerate regime).

Collisionless sound and comparison with experiment (III)

In this way, the coupled equations for the real and imaginary part of the zero-sound velocity read

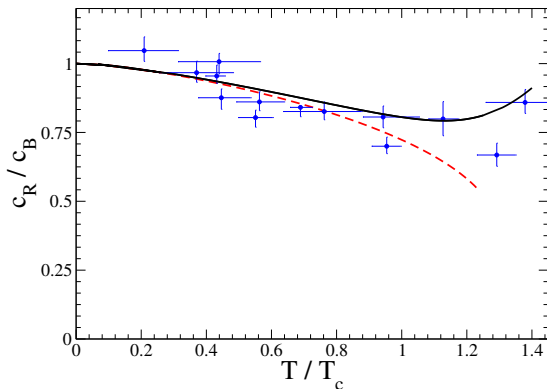
$$1 + \frac{\tilde{g}_{2D} k_B T}{2\pi} \left[\frac{2}{m_{CR}^2 - 2\tilde{\mu}} + \frac{\sqrt{m_{CR}^2}}{(m_{CR}^2 - 2\tilde{\mu})^{3/2}} \ln \left(\frac{\sqrt{m_{CR}^2 - 2\tilde{\mu}} - \sqrt{m_{CR}^2}}{\sqrt{m_{CR}^2 - 2\tilde{\mu}} + \sqrt{m_{CR}^2}} \right) \right] + \tilde{g}_{2D} k_B T c_I \frac{m_{CR}^2 + \tilde{\mu}}{\sqrt{m}(m_{CR}^2 - 2\tilde{\mu})^{5/2}} = 0, \quad (16)$$

$$c_I = - \frac{\frac{c_R}{\sqrt{m}(m_{CR}^2 - 2\tilde{\mu})^{3/2}}}{\frac{6 c_R}{(m_{CR}^2 - 2\tilde{\mu})^2} + \frac{2(m_{CR}^2 + \tilde{\mu})}{\sqrt{m}(m_{CR}^2 - 2\tilde{\mu})^{5/2}} \log \left(\frac{\sqrt{m_{CR}^2 - 2\tilde{\mu}} - \sqrt{m_{CR}^2}}{\sqrt{m_{CR}^2 - 2\tilde{\mu}} + \sqrt{m_{CR}^2}} \right)}, \quad (17)$$

where

$$\tilde{g}_{2D} = \frac{m}{\hbar^2} g_{2D} = \sqrt{8\pi} \frac{a_s}{a_z}. \quad (18)$$

Collisionless sound and comparison with experiment (IV)

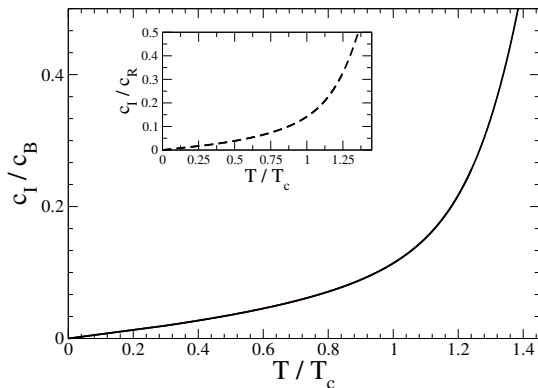


Sound velocity c_R in units of $c_B = \sqrt{g_{2D}n/m}$ vs T/T_c for $\tilde{g}_{2D} = 0.16$. The **solid black line** represents our theoretical prediction¹⁰ while the blue dots are the experimental data obtained at ENS Paris.¹¹ The **red dashed line** is obtained by our equations with $c_I = 0$. Here $T_c = 0.13 T_B$.

¹⁰A. Cappellaro, F. Toigo, and LS, Phys. Rev. A **98**, 043605 (2018).

¹¹J. L. Ville *et al.*, Phys. Rev. Lett. **121**, 145301 (2018).

Collisionless sound and comparison with experiment (V)



Imaginary part c_I of the sound velocity in units of $c_B = \sqrt{g_{2D}n/m}$ as a function of the scaled temperature T/T_c for $\tilde{g}_{2D} \simeq 0.16$.

Inset: Ratio c_I/c_R between the imaginary and the real part of the sound velocity c as a function of the temperature.

Conclusions

- We have analyzed the sound propagation in collisionless bosonic gases assuming a 2D configuration.
- By solving the linearized **2D Boltzmann-Landau-Vlasov equation** in the degenerate regime, we have:
 - derived two coupled algebraic equations for the real and imaginary part of the sound velocity;
 - compared our theoretical results with the recent experiment at ENS Paris;
 - found a very good agreement between our theory and the experiment.
- Our theoretical analysis strongly suggests that the density perturbation used in the experiment has excited the "**bosonic zero sound**", i.e. the sound of a collisionless bosonic fluid.
- The two-fluid hydrodynamics of Landau-Khalatnikov describes correctly only the collisional regime.

Thank you for attention!

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