

Vortices and Antivortices in Two-Dimensional Ultracold Fermi Gases

Luca Salasnich

Dipartimento di Fisica e Astronomia "Galileo Galilei" and CNISM, Università di Padova
INO-CNR, Research Unit of Sesto Fiorentino, Consiglio Nazionale delle Ricerche

Natal, November 14, 2016

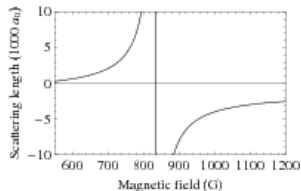
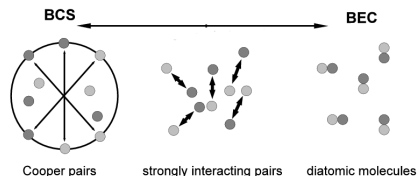
Work done in collaboration with
Giacomo Bighin and Flavio Toigo

Summary

- BCS-BEC crossover in 2D
- Quantum fluctuations in 2D
- Vortices and antivortices in 2D
- Conclusions

BCS-BEC crossover in 2D (I)

In 2004 the **3D BCS-BEC crossover** has been observed with **ultracold gases made of two-component fermionic ^{40}K or ^6Li alkali-metal atoms**.¹



This crossover is obtained by using a Fano-Feshbach resonance to change the 3D s-wave scattering length a_s of the inter-atomic potential

$$a_s = a_{bg} \left(1 + \frac{\Delta_B}{B - B_0} \right), \quad (1)$$

where B is the external magnetic field.

¹C.A. Regal et al., PRL **92**, 040403 (2004); M.W. Zwierlein et al., PRL **92**, 120403 (2004); J. Kinast et al., PRL **92**, 150402 (2004).

BCS-BEC crossover in 2D (II)

Recently also the [2D BEC-BEC crossover](#) has been achieved experimentally² with a **Fermi gas of two-component ⁶Li atoms**. In 2D attractive fermions always form biatomic molecules with bound-state energy

$$\epsilon_B \simeq \frac{\hbar^2}{ma_s^2}, \quad (2)$$

where a_s is the 2D s-wave scattering length, which is experimentally tuned by a **Fano-Feshbach resonance**.

The [fermionic single-particle spectrum](#) is given by

$$E_{sp}(k) = \sqrt{\left(\frac{\hbar^2 k^2}{2m} - \mu\right)^2 + \Delta^2}, \quad (3)$$

where Δ is the [energy gap](#) and μ is the [chemical potential](#): $\mu > 0$ corresponds to the BCS regime while $\mu < 0$ corresponds to the BEC regime. Moreover, in the deep BEC regime $\mu \rightarrow -\epsilon_B/2$.

²V. Makhalov et al. PRL **112**, 045301 (2014); M.G. Ries et al., PRL **114**, 230401 (2015); I. Boettcher et al., PRL **116**, 045303 (2016).

BCS-BEC crossover in 2D (III)

To study the 2D BCS-BEC crossover we adopt the formalism of **functional integration**³. The **partition function** \mathcal{Z} of the uniform system with fermionic fields $\psi_s(\mathbf{r}, \tau)$ at temperature T , in a D -dimensional volume L^D , and with chemical potential μ reads

$$\mathcal{Z} = \int \mathcal{D}[\psi_s, \bar{\psi}_s] \exp \left\{ -\frac{S}{\hbar} \right\}, \quad (4)$$

where ($\beta \equiv 1/(k_B T)$ with k_B Boltzmann's constant)

$$S = \int_0^{\hbar\beta} d\tau \int_{L^D} d^D \mathbf{r} \mathcal{L} \quad (5)$$

is the **Euclidean action functional** with **Lagrangian density**

$$\mathcal{L} = \bar{\psi}_s \left[\hbar \partial_\tau - \frac{\hbar^2}{2m} \nabla^2 - \mu \right] \psi_s + \mathbf{g} \bar{\psi}_\uparrow \bar{\psi}_\downarrow \psi_\downarrow \psi_\uparrow \quad (6)$$

where **\mathbf{g} is the attractive strength ($\mathbf{g} < 0$) of the s-wave coupling.**

³N. Nagaosa, Quantum Field Theory in Condensed Matter Physics (Springer, 1999)

BCS-BEC crossover in 2D (IV)

Through the usual **Hubbard-Stratonovich transformation** the Lagrangian density \mathcal{L} , quartic in the fermionic fields, can be rewritten as a quadratic form by introducing the **auxiliary complex scalar field** $\Delta(\mathbf{r}, \tau)$. In this way the effective Euclidean Lagrangian density reads

$$\mathcal{L}_e = \bar{\psi}_s \left[\hbar \partial_\tau - \frac{\hbar^2}{2m} \nabla^2 - \mu \right] \psi_s + \bar{\Delta} \psi_\downarrow \psi_\uparrow + \Delta \bar{\psi}_\uparrow \bar{\psi}_\downarrow - \frac{|\Delta|^2}{\mathbf{g}}. \quad (7)$$

We investigate the effect of fluctuations of **the gap field** $\Delta(\mathbf{r}, t)$ around its mean-field value Δ_0 which may be taken to be real. For this reason we set

$$\Delta(\mathbf{r}, \tau) = \Delta_0 + \eta(\mathbf{r}, \tau), \quad (8)$$

where $\eta(\mathbf{r}, \tau)$ is the complex field which describes pairing fluctuations.

BCS-BEC crossover in 2D (V)

In particular, we are interested in **the grand potential** Ω , given by

$$\Omega = -\frac{1}{\beta} \ln(\mathcal{Z}) \simeq -\frac{1}{\beta} \ln(\mathcal{Z}_{mf} \mathcal{Z}_g) = \Omega_{mf} + \Omega_B, \quad (9)$$

where

$$\mathcal{Z}_{mf} = \int \mathcal{D}[\psi_s, \bar{\psi}_s] \exp \left\{ -\frac{S_e(\psi_s, \bar{\psi}_s, \Delta_0)}{\hbar} \right\} \quad (10)$$

is the mean-field partition function and

$$\mathcal{Z}_g = \int \mathcal{D}[\psi_s, \bar{\psi}_s] \mathcal{D}[\eta, \bar{\eta}] \exp \left\{ -\frac{S_g(\psi_s, \bar{\psi}_s, \eta, \bar{\eta}, \Delta_0)}{\hbar} \right\} \quad (11)$$

is the partition function of Gaussian pairing fluctuations.

Quantum fluctuations in 2D (I)

One finds that in the gas of paired fermions there are **two kinds of elementary excitations**: **fermionic single-particle excitations** with energy

$$E_{sp}(k) = \sqrt{\left(\frac{\hbar^2 k^2}{2m} - \mu\right)^2 + \Delta_0^2}, \quad (12)$$

where Δ_0 is the pairing gap, and **bosonic collective excitations** with energy (at low-momenta and in the BEC regime)

$$E_{col}(q) = \sqrt{\frac{\hbar^2 q^2}{2m} \left(\lambda \frac{\hbar^2 q^2}{2m} + 2 m c_s^2 \right)}, \quad (13)$$

where λ is the first correction to the familiar phonon dispersion $E_{col}(q) \simeq c_s \hbar q$ and c_s is the sound velocity. Notice that both λ and c_s depend on the chemical potential μ .

Quantum fluctuations in 2D (II)

In the analysis of the **two-dimensional attractive Fermi gas** one must remember that, contrary to the 3D case, **2D realistic interatomic attractive potentials have always a bound state**. In particular⁴, the binding energy $\epsilon_B > 0$ of two fermions can be written in terms of the positive 2D fermionic scattering length a_s as

$$\epsilon_B = \frac{4}{e^{2\gamma}} \frac{\hbar^2}{m a_s^2}, \quad (14)$$

where $\gamma = 0.577\dots$ is the Euler-Mascheroni constant. Moreover, the attractive (negative) interaction strength \mathbf{g} of s-wave pairing is related to the binding energy $\epsilon_B > 0$ of a fermion pair in vacuum by the expression⁵

$$-\frac{1}{\mathbf{g}} = \frac{1}{2L^2} \sum_{\mathbf{k}} \frac{1}{\frac{\hbar^2 k^2}{2m} + \frac{1}{2}\epsilon_B}. \quad (15)$$

⁴C. Mora and Y. Castin, 2003, PRA **67**, 053615.

⁵M. Randeria, J-M. Duan, and L-Y. Shieh, PRL **62**, 981 (1989).

Quantum fluctuations in 2D (III)

In the **2D BCS-BEC crossover**, at zero temperature ($T = 0$) the mean-field grand potential Ω_{mf} can be written as⁶ ($\epsilon_B > 0$)

$$\Omega_{mf} = -\frac{mL^2}{2\pi\hbar^2} \left(\mu + \frac{1}{2}\epsilon_B\right)^2. \quad (16)$$

Using

$$n = -\frac{1}{L^2} \frac{\partial \Omega_{mf}}{\partial \mu} \quad (17)$$

one immediately finds the chemical potential μ as a function of the number density $n = N/L^2$, i.e.

$$\mu = \frac{\pi\hbar^2}{m} n - \frac{1}{2}\epsilon_B. \quad (18)$$

In the BCS regime, where $\epsilon_B \ll \epsilon_F$ with $\epsilon_F = \pi\hbar^2 n/m$, one finds $\mu \simeq \epsilon_F > 0$ while in the BEC regime, where $\epsilon_B \gg \epsilon_F$ one has $\mu \simeq -\epsilon_B/2 < 0$.

⁶M. Randeria, J-M. Duan, and L-Y. Shieh, PRL **62**, 981 (1989).

Quantum fluctuations in 2D (IV)

In the deep BEC regime of the **2D BCS-BEC crossover**, where the chemical potential μ becomes negative, performing **regularization of zero-point fluctuations** we have recently found⁷ that the zero-temperature grand potential (including **bosonic excitations**) is

$$\Omega = -\frac{mL^2}{64\pi\hbar^2} \left(\mu + \frac{1}{2}\epsilon_B\right)^2 \ln \left(\frac{\epsilon_B}{2\left(\mu + \frac{1}{2}\epsilon_B\right)} \right). \quad (19)$$

This is exactly Popov's equation of state of 2D Bose gas with chemical potential $\mu_B = 2\left(\mu + \epsilon_B/2\right)$ and mass $m_B = 2m$. In this way we have identified the two-dimensional scattering length a_B of composite bosons as

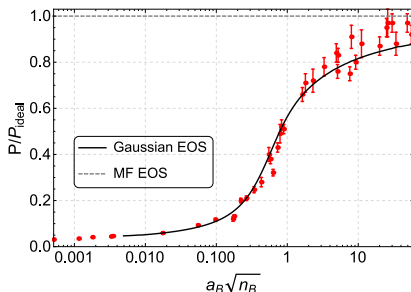
$$a_B = \frac{1}{2^{1/2}e^{1/4}} a_s. \quad (20)$$

The value $a_B/a_s = 1/(2^{1/2}e^{1/4}) \simeq 0.551$ is in full agreement with $a_B/a_s = 0.55(4)$ obtained by Monte Carlo calculations⁸.

⁷LS and F. Toigo, PRA **91**, 011604(R) (2015).

⁸G. Bertaina and S. Giorgini, PRL **106**, 110403 (2011).

Quantum fluctuations in 2D (V)



Scaled pressure $P/(2P_{ideal})$ as a function of the bosonic gas parameter $a_B n_B^{1/2}$, where $P_{ideal} = 2\pi\hbar^2 n_B^2 / m_B$ is the pressure of an ideal 2D gas with $m_B = 2m$ the mass of each bosonic particle (made of two fermions with mass m), a_B is the s-wave scattering length of bosons, and $n_B = n/2$ is the bosonic 2D density (with n the fermionic density). Filled squares with error bars are experimental data of Makhalov *et al.*⁹. Solid line is obtained with our renormalized Gaussian theory¹⁰

⁹V. Makhalov et al. PRL **112**, 045301 (2014)

¹⁰G. Bighin and LS, PRB **93**, 014519 (2016).

Vortices and antivortices in 2D (I)

The low-energy Hamiltonian of a 2D fermionic superfluid can be recast as that of an effective continuous 2D XY model

$$H = \frac{J(T)}{2} \int d^2\mathbf{r} (\nabla\theta(\mathbf{r}))^2, \quad (21)$$

where $\theta(\mathbf{r})$ is the phase angle of the pairing field $\Delta(\mathbf{r}) = |\Delta(\mathbf{r})|e^{i\theta(\mathbf{r})}$ and

$$J(T) = \frac{\hbar^2}{4m} n_s(T) \quad (22)$$

is the phase stiffness and $n_s(T)$ is the superfluid density. The compactness of the phase angle $\theta(\mathbf{r})$ implies that

$$\oint \nabla\theta(\mathbf{r}) \cdot d\mathbf{r} = 2\pi q, \quad (23)$$

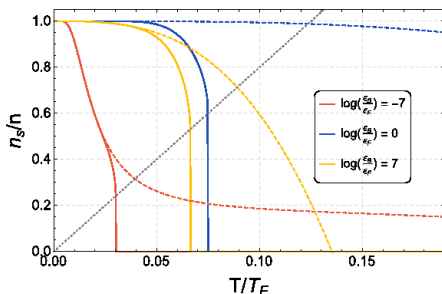
where q is the integer number associated to **quantum vortices** ($q > 0$) and **antivortices** ($q < 0$), which **renormalize**¹¹ $n_s(T)$ and $J(T)$.

¹¹J.M. Kosterlitz and D.J. Thouless, J. Phys. C **6**, 1181 (1973).

Vortices and antivortices in 2D (II)

In our approach the (bare) superfluid density is given by

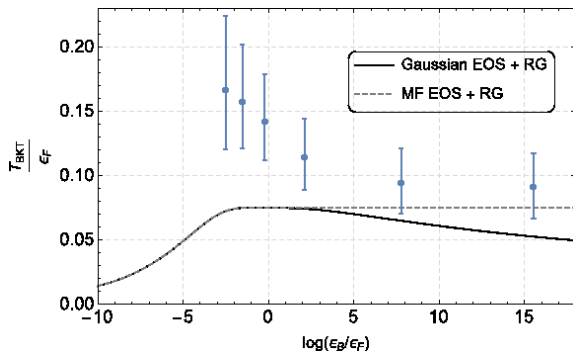
$$n_s(T) = n - \beta \int \frac{d^2k}{(2\pi)^2} k^2 \frac{e^{\beta E_{sp}(k)}}{(e^{\beta E_{sp}(k)} + 1)^2} - \frac{\beta}{2} \int \frac{d^2q}{(2\pi)^2} q^2 \frac{e^{\beta E_{col}(q)}}{(e^{\beta E_{col}(q)} - 1)^2} \quad (24)$$



The **Berezinskii-Kosterlitz-Thouless critical temperature** T_{BKT} is determined by the jump of the **renormalized superfluid density** $n_{s,r}(T)$, which derived from **Kosterlitz's renormalization-group equations**.¹²

¹²J.M. Kosterlitz and D.J. Thouless, J. Phys. C **6**, 1181 (1973).

Vortices and antivortices in 2D (III)

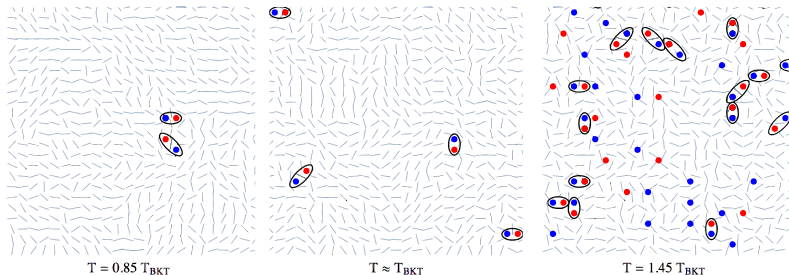


Theoretical predictions for the [Berezinskii-Kosterlitz-Thouless critical temperature](#) T_{BKT} (at which [vortex-antivortex pairs](#) unbind) compared¹³ to recent experimental observation¹⁴ (circles with error bars).

¹³G. Bighin and LS, PRB **93**, 014519 (2016); G. Bighin and LS, submitted.

¹⁴P.A. Murthy et al., PRL **115**, 010401 (2015).

Vortices and antivortices in 2D (IV)

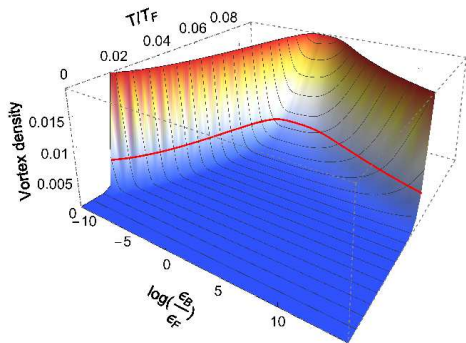


Monte Carlo simulation of the **vortex-antivortex unbinding** transition at T_{BKT} .¹⁵ Arrows denote the local gradient of the phase field, i.e. $\nabla\theta(\mathbf{r})$, while the red and blue dots correspond to vortices with ± 1 topological charge, i.e. **vortices** and **antivortices**.

A black ellipse is drawn around bound vortices, to guide the eye in visualizing the vortex unbinding transition. The bound state energy is $\log(\epsilon_B/\epsilon_F) = 5$.

¹⁵Discrete XY model in a 64×64 lattice with Wolff algorithm. G. Bighin and LS, submitted.

Vortices and antivortices in 2D (V)



Vortex density as a function of the fermion-fermion interaction strength and of the temperature, as calculated from Monte Carlo simulations.¹⁶ The blue line denotes the BKT critical temperature.

¹⁶G. Bighin and LS, submitted.

Conclusions

- The **regularization of zero-point energy**¹⁷ gives remarkable **beyond-mean-field effects** for composite bosons in the 2D BCS-BEC crossover at zero temperature:
 - logarithmic behavior of the equation of state
 - Bose-Bose scattering length a_B vs Fermi-Fermi scattering length a_s
 - speed of first sound (and also second sound)
- Also at finite temperature **beyond-mean-field effects**, with the inclusion of **quantized vortices and antivortices**, become relevant in the strong-coupling regime of 2D BCS-BEC crossover:
 - superfluid density n_s
 - critical temperature T_{BKT}
 - proliferation of quantized vortices

¹⁷For a very recent **comprehensive review** see:

LS and F. Toigo, Zero-Point Energy of Ultracold Atoms, Physics Reports 640, 1 (2016).

Thank you for your attention!

Main sponsors: PRIN-MIUR, UNIPD.