Quantum fluctuations and vortex-antivortex unbinding in the 2D BCS-BEC crossover

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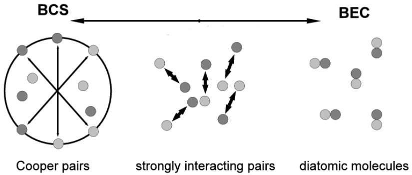
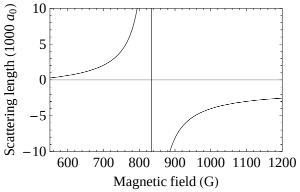
Summary

- BCS-BEC crossover in 2D
- Quantum fluctuations in 2D
- New results for 2D BCS-BEC crossover
- Conclusions

BCS-BEC crossover in 2D (I)

In 2004 the 3D BCS-BEC crossover has been observed with **ultracold gases made of two-component fermionic ^{40}K or ^{6}Li alkali-metal atoms.**[1]





This crossover is obtained by using a Fano-Feshbach resonance to change the 3D s-wave scattering length a_s of the inter-atomic potential

$$a_s = a_{bg}\left(1 + \frac{\Delta_B}{B - B_0}\right) , \qquad (1)$$

where B is the external magnetic field.

[1] C.A. Regal et al., PRL **92**, 040403 (2004); M.W. Zwierlein et al., PRL **92**, 120403 (2004); J. Kinast et al., PRL **92**, 150402 (2004).

BCS-BEC crossover in 2D (II)

Recently also the 2D BEC-BEC crossover has been achieved experimentally[2] with a **Fermi gas of two-component ^{6}Li atoms**. In 2D attractive fermions always form biatomic molecules with bound-state energy

$$\epsilon_B \simeq \frac{\hbar^2}{m {a_s}^2} , \tag{2}$$

where a_s is the 2D s-wave scattering length, which is experimentally tuned by a Fano-Feshbach resonance.

The fermionic single-particle spectrum is given by

$$E_{sp}(k) = \sqrt{\left(\frac{\hbar^2 k^2}{2m} - \mu \right)^2 + \Delta^2} , \tag{3}$$

where Δ is the energy gap and μ is the chemical potential: $\mu > 0$ corresponds to the BCS regime while $\mu < 0$ corresponds to the BEC regime. Moroever, in the deep BEC regime $\mu \to -\epsilon_B/2$.

[2] V. Makhalov et al. PRL **112**, 045301 (2014); M.G. Ries et al., PRL **114**, 230401 (2015); I. Boettcher et al., PRL **116**, 045303 (2016).

BCS-BEC crossover in 2D (III)

To study the 2D BCS-BEC crossover we adopt the formalism of functional integration[3]. The partition function $\mathcal{Z}$ of the uniform system with fermionic fields $\psi_s(\mathbf{r},\tau)$ at temperature T, in a D-dimensional volume L^D, and with chemical potential μ reads

$$\mathcal{Z} = \int \mathcal{D}[\psi_s, \bar{\psi}_s] \ \exp\left\{ -\frac{S}{\hbar} \right\} , \qquad (4)$$

where ($\beta \equiv 1/(k_B T)$ with k_B Boltzmann's constant)

$$S = \int_0^{\hbar\beta} d\tau \int_{L^D} d^D\mathbf{r} \ \mathcal{L} \qquad (5)$$

is the Euclidean action functional with Lagrangian density

$$\mathcal{L} = \bar{\psi}_s \left[\hbar\partial_\tau - \frac{\hbar^2}{2m}\nabla^2 - \mu \right] \psi_s + \mathbf{g}\, \bar{\psi}_\uparrow \, \bar{\psi}_\downarrow \, \psi_\downarrow \, \psi_\uparrow \qquad (6)$$

where $\mathbf{g}$ is the attractive strength ($\mathbf{g} < 0$) of the s-wave coupling.

[3] N. Nagaosa, Quantum Field Theory in Condensed Matter Physics (Springer, 1999)

BCS-BEC crossover in 2D (IV)

Through the usual Hubbard-Stratonovich transformation the Lagrangian density $\mathcal{L}$, quartic in the fermionic fields, can be rewritten as a quadratic form by introducing the auxiliary complex scalar field $\Delta(\mathbf{r},\tau)$. In this way the effective Euclidean Lagrangian density reads

$$\mathcal{L}_e = \bar{\psi}_s \left[\hbar \partial_\tau - \frac{\hbar^2}{2m} \nabla^2 - \mu \right] \psi_s + \bar{\Delta}\, \psi_\downarrow\, \psi_\uparrow + \Delta \bar{\psi}_\uparrow\, \bar{\psi}_\downarrow - \frac{|\Delta|^2}{\mathbf{g}} \, . \qquad (7)$$

We investigate the effect of fluctuations of the gap field $\Delta(\mathbf{r}, t)$ around its mean-field value Δ_0 which may be taken to be real. For this reason we set

$$\Delta(\mathbf{r},\tau) = \Delta_0 + \eta(\mathbf{r},\tau) \, , \qquad (8)$$

where $\eta(\mathbf{r},\tau)$ is the complex field which describes pairing fluctuations.

BCS-BEC crossover in 2D (V)

In particular, we are interested in the grand potential Ω, given by

$$\Omega = -\frac{1}{\beta} \ln(\mathcal{Z}) \simeq -\frac{1}{\beta} \ln(\mathcal{Z}_{mf}\mathcal{Z}_g) = \Omega_{mf} + \Omega_B \ , \tag{9}$$

where

$$\mathcal{Z}_{mf} = \int \mathcal{D}[\psi_s, \bar{\psi}_s] \exp\left\{-\frac{S_e(\psi_s, \bar{\psi}_s, \Delta_0)}{\hbar}\right\} \tag{10}$$

is the mean-field partition function and

$$\mathcal{Z}_g = \int \mathcal{D}[\psi_s, \bar{\psi}_s] \mathcal{D}[\eta, \bar{\eta}] \ \exp\left\{-\frac{S_g(\psi_s, \bar{\psi}_s, \eta, \bar{\eta}, \Delta_0)}{\hbar}\right\} \tag{11}$$

is the partition function of Gaussian pairing fluctuations.

Quantum fluctuations in 2D (I)

One finds that in the gas of paired fermions there are two kinds of elementary excitations: fermionic single-particle excitations with energy

$$E_{sp}(k) = \sqrt{\left(\frac{\hbar^2 k^2}{2m} - \mu\right)^2 + \Delta_0^2} \,, \tag{12}$$

where Δ_0 is the pairing gap, and bosonic collective excitations with energy

$$E_{col}(q) = \sqrt{\frac{\hbar^2 q^2}{2m}\left(\lambda \, \frac{\hbar^2 q^2}{2m} + 2 \; m \; c_s^2\right)} \,, \tag{13}$$

where λ is the first correction to the familiar low-momentum phonon dispersion $E_{col}(q) \simeq c_s \hbar q$ and c_s is the sound velocity. Notice that both λ and c_s depend on the chemical potential μ.

Quantum fluctuations in 2D (II)

Moreover, at the Gaussian level, the total grand potential reads

$$\Omega = \Omega_{mf} + \Omega_g \ , \tag{14}$$

where

$$\Omega_{mf} = \Omega_0 + \Omega_F^{(0)} + \Omega_F^{(T)} \tag{15}$$

is the mean-field grand potential with

$$\Omega_0 = -\frac{\Delta_0^2}{\mathbf{g}} L^D \tag{16}$$

the grand potential of the order parameter Δ_0,

$$\Omega_F^{(0)} = -\sum_{\mathbf{k}} \left(E_{sp}(k) - \frac{\hbar^2 k^2}{2m} + \mu \right) \tag{17}$$

the zero-point energy of fermionic single-particle excitations,

$$\Omega_F^{(T)} = \frac{2}{\beta} \sum_{\mathbf{k}} \ln\left(1 + e^{-\beta\, E_{sp}(k)}\right) \tag{18}$$

the finite-temperature grand potential of the fermionic single-particle excitations.

Quantum fluctuations in 2D (III)

The grand-potential of bosonic Gaussian fluctuations reads

$$\Omega_g = \Omega_{g,B}^{(0)} + \Omega_{g,B}^{(T)} \,, \tag{19}$$

where

$$\Omega_{g,B}^{(0)} = \frac{1}{2} \sum_{\mathbf{q}} E_{col}(q) \tag{20}$$

is the zero-point energy of bosonic collective excitations and

$$\Omega_{g,B}^{(T)} = \frac{1}{\beta} \sum_{\mathbf{q}} \ln\left(1 - e^{-\beta\, E_{col}(q)}\right) \tag{21}$$

is the finite-temperature grand potential of the bosonic collective excitations.

Both $\Omega_F^{(0)}$ and $\Omega_{g,B}^{(0)}$ are ultraviolet divergent in any dimension D ($D = 1, 2, 3$) and the regularization of these divergent terms is complicated by the fact that one also must take into account the BCS-BEC crossover.

New results for 2D BCS-BEC crossover (I)

In the analysis of the **two-dimensional attractive Fermi gas** one must remember that, contrary to the 3D case, 2D realistic interatomic attractive potentials have always a bound state. In particular[4], the binding energy $\epsilon_B > 0$ of two fermions can be written in terms of the positive 2D fermionic scattering length a_s as

$$\epsilon_B = \frac{4}{e^{2\gamma}} \frac{\hbar^2}{m{a_s}^2} \; , \tag{22}$$

where $\gamma = 0.577...$ is the Euler-Mascheroni constant. Moreover, the attractive (negative) interaction strength $\mathbf{g}$ of s-wave pairing is related to the binding energy $\epsilon_B > 0$ of a fermion pair in vacuum by the expression[5]

$$-\frac{1}{\mathbf{g}} = \frac{1}{2L^2} \sum_{\mathbf{k}} \frac{1}{\frac{\hbar^2 k^2}{2m} + \frac{1}{2}\epsilon_B} \; . \tag{23}$$

[4] C. Mora and Y. Castin, 2003, PRA **67**, 053615.

[5] M. Randeria, J-M. Duan, and L-Y. Shieh, PRL **62**, 981 (1989).

New results for 2D BCS-BEC crossover (II)

In the **2D BCS-BEC crossover**, at zero temperature ($T = 0$) the mean-field grand potential Ω_{mf} can be written as[6] ($\epsilon_B > 0$)

$$\Omega_{mf} = -\frac{mL^2}{2\pi\hbar^2}(\mu + \frac{1}{2}\epsilon_B)^2 \ . \tag{24}$$

Using

$$n = -\frac{1}{L^2}\frac{\partial \Omega_{mf}}{\partial \mu} \tag{25}$$

one immediately finds the chemical potential μ as a function of the number density $n = N/L^2$, i.e.

$$\mu = \frac{\pi\hbar^2}{m} n - \frac{1}{2}\epsilon_B \ . \tag{26}$$

In the BCS regime, where $\epsilon_B \ll \epsilon_F$ with $\epsilon_F = \pi\hbar^2 n/m$, one finds $\mu \simeq \epsilon_F > 0$ while in the BEC regime, where $\epsilon_B \gg \epsilon_F$ one has $\mu \simeq -\epsilon_B/2 < 0$.

[6] M. Randeria, J-M. Duan, and L-Y. Shieh, PRL **62**, 981 (1989).

New results for 2D BCS-BEC crossover (III)

In the deep BEC regime of the **2D BCS-BEC crossover**, where the chemical potential μ becomes negative, performing regularization of zero-point fluctuations we have recently found[7] that the zero-temperature grand potential (including bosonic excitations) is

$$\Omega = -\frac{mL^2}{64\pi\hbar^2}(\mu + \frac{1}{2}\epsilon_B)^2 \, \ln\left(\frac{\epsilon_B}{2(\mu+\frac{1}{2}\epsilon_B)}\right) . \tag{27}$$

This is exactly Popov's equation of state of 2D Bose gas with chemical potential $\mu_B = 2(\mu + \epsilon_B/2)$ and mass $m_B = 2m$. In this way we have identified the two-dimensional scattering length a_B of composite bosons as

$$\boxed{a_B = \frac{1}{2^{1/2}e^{1/4}} \, a_s \, .} \tag{28}$$

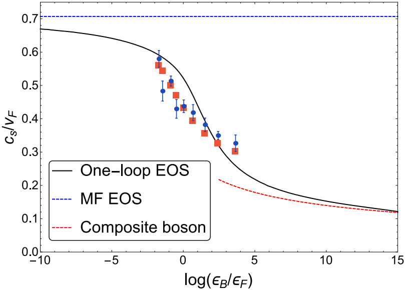
The value $a_B/a_s = 1/(2^{1/2}e^{1/4}) \simeq 0.551$ is in full agreement with $a_B/a_s = 0.55(4)$ obtained by Monte Carlo calculations[8].

[7] LS and F. Toigo, PRA **91**, 011604(R) (2015).
[8] G. Bertaina and S. Giorgini, PRL **106**, 110403 (2011).

New results for 2D BCS-BEC crossover (IV)

At zero temperature we compare[9] the first sound velocity

$$c_s = \sqrt{\frac{n}{m}\frac{\partial \mu}{\partial n}} = \sqrt{-\frac{n}{m}\left(\frac{1}{L^2}\frac{\partial^2 \Omega(\mu)}{\partial \mu^2}\right)^{-1}} \,. \tag{29}$$



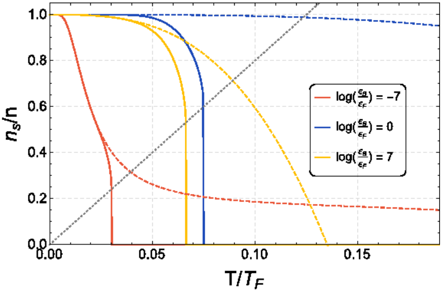
with available experimental data[10] (blue circles and red squares).

[9] G. Bighin and LS, PRB **93**, 014519 (2016).
[10] N. Luick, M.Sc. Thesis, Supervisors: E. Moritz and L. Mathey, University of Hamburg (2014).

New results for 2D BCS-BEC crossover (V)

The Berezinskii-Kosterlitz-Thouless critical temperature T_{BKT} is determined by the jump of the renormalized superfluid density $n_{s,r}(T)$, derived[11] starting from the bare superfluid density

$$n_s(T) = n - \beta \int \frac{d^2k}{(2\pi)^2} k^2 \frac{e^{\beta E_{sp}(k)}}{(e^{\beta E_{sp}(k)}+1)^2} - \frac{\beta}{2} \int \frac{d^2q}{(2\pi)^2} q^2 \frac{e^{\beta E_{col}(q)}}{(e^{\beta E_{col}(q)}-1)^2} \quad (30)$$



and using **Kosterlitz's renormalization-group equations**.[12]

[11] G. Bighin and LS, in preparation.

[12] J.M. Kosterlitz and D.J. Thouless, J. Phys. C **6**, 1181 (1973).

New results for 2D BCS-BEC crossover (VI)

In fact the low-energy Hamiltonian of a fermionic superfluid can be recast[13] as that of an effective continuous 2D XY model

$$H = \frac{J(T)}{2} \int d^2\mathbf{r} \left(\nabla\theta(\mathbf{r})\right)^2 \ , \qquad (31)$$

where $\theta(\mathbf{r})$ is the phase angle of the pairing field $\Delta(\mathbf{r}) = |\Delta(\mathbf{r})| e^{i\theta(\mathbf{r})}$ and

$$J(T) = \frac{\hbar^2}{4m} n_s(T) \qquad (32)$$

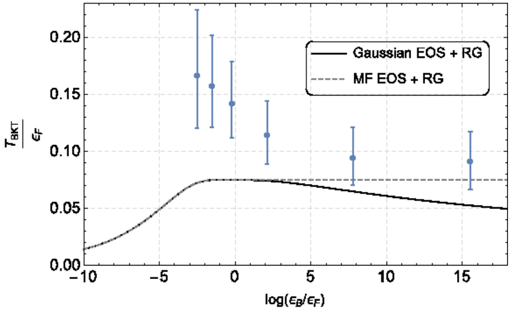
is the phase stiffness. The compactness of the phase angle $\theta(\mathbf{r})$ implies that

$$\oint \nabla\theta(\mathbf{r}) \cdot d\mathbf{r} = 2\pi q \ , \qquad (33)$$

where q is the integer number associated to quantum vortices ($q > 0$) and antivortices ($q < 0$), which renormalize[14] the phase stiffness and consequently also the superfluid density.

[13] E. Babaev and H. Kleinert, Phys. Rev. B **59**, 12083 (1999).
[14] J.M. Kosterlitz and D.J. Thouless, J. Phys. C **6**, 1181 (1973).

New results for 2D BCS-BEC crossover (VII)



Theoretical predictions for the Berezinskii-Kosterlitz-Thouless critical temperature T_{BKT} (at which vortex-antivortex pairs unbind) compared[15] to recent experimental observation[16] (circles with error bars).

[15] G. Bighin and LS, PRB **93**, 014519 (2016); G. Bighin and LS, in preparation.
[16] P.A. Murthy et al., PRL **115**, 010401 (2015).

Conclusions

- The regularization of zero-point energy[17] gives remarkable beyond-mean-field effects for composite bosons in the 2D BCS-BEC crossover at zero temperature:
  – logarithmic behavior of the equation of state
  – Bose-Bose scattering length a_B vs Fermi-Fermi scattering length a_s
  – speed of first sound (and also second sound)
- Also at finite temperature beyond-mean-field effects, with the inclusion of quantized vortices and antivortices, become relevant in the strong-coupling regime of 2D BCS-BEC crossover:
  – superfluid density n_s
  – critical temperature T_{BKT}

[17] For a very recent comprehensive review see:
L. Salasnich and F. Toigo, Zero-Point Energy of Ultracold Atoms, arXiv: 1606.03699, Physics Reports, in press.

Acknowledgements

Thank you for your attention!

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