Composite bosons in the 2D BCS-BEC crossover from Gaussian fluctuations

Luca Salasnich and Flavio Toigo

Department of Physics and Astronomy “Galileo Galilei”, University of Padua, Italy

Trieste, October 28, 2014

Grand potential in 2D BCS-BEC crossover

Within our beyond-mean-field approach the zero-temperature grand potential of the **2D Fermi superfluid** in the BCS-BEC crossover is given by

\[ \Omega = \Omega_{mf} + \Omega_{g} , \]  

(1)

where

\[ \Omega_{mf} = - \frac{mL^2}{2\pi\hbar^2} (\mu + \frac{1}{2}\epsilon_{b})^2 \]  

(2)

is the mean-field term of single-particle fermionic elementary excitations, with \( \mu \) the chemical potential, \( \epsilon_B \) the binding energy of paired fermions, and \( L^2 \) the area. The beyond-mean-field Gaussian term

\[ \Omega_{g} = \frac{1}{2} \sum_{q} \sqrt{\frac{\hbar^2 q^2}{2m}} \left( \lambda \frac{\hbar^2 q^2}{2m} + 2m c_s^2 \right) \]  

(3)

is the zero-point energy of collective bosonic elementary excitations, where \( c_s \) is the sound velocity and \( \lambda \) the quartic correction of the familiar low-momentum dispersion \( \epsilon_{col}(q) = c_s q \).
$\Omega_g$ is *divergent* but we regularize it by using dimensional regularization. In particular, in the BEC regime ($\lambda = 1/4$) we find

$$\Omega = -\frac{m_B L^2}{8\pi \hbar^2 \mu_B^2} \ln \left( \frac{\epsilon_b}{\mu_B} \right). \quad (4)$$

where $\mu_B = 2(\mu + \epsilon_b/2)$ is the chemical potential of composite bosons. In terms of the fermionic scattering length $a_F$ we have $\epsilon_b = \frac{4}{e^{2\gamma}} \frac{\hbar^2}{ma_F^2}$, where $\gamma = 0.577$. In this way Eq. (4) becomes exactly the Popov’s 2D equation of state\(^1\) of bosons with scattering length $a_B$, provided that

$$a_B = \frac{1}{2^{1/2}e^{1/4}} a_F = 0.551...a_F, \quad (5)$$

in good agreement with Monte Carlo\(^2\) and four-body scattering\(^3\).

---