Density functional of the Fermi gas in the BCS-BEC crossover

Luca Salasnich

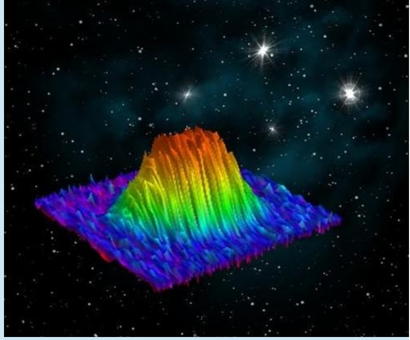
Department of Physics and Astronomy "G.Galilei"
University of Padua, Italy

Budapest, November 7, 2013

In collaboration with:

S.K. Adhikari (Sao Paulo), N. Manini (Milan) F. Ancilotto, F. Toigo (Padua)

Plan of the talk



- Cold Fermi gas superfluidity in the BCS-BEC crossover

- Extended Superfluid Hydrodynamics or TD ETF
  description of strongly interacting Fermi gas at T=0

- Validation of the model : Josephson current

- Long-time dynamics of the collision between Fermi clouds and shock wave formations:
  experiment vs. theory
  dispersion vs. dissipation

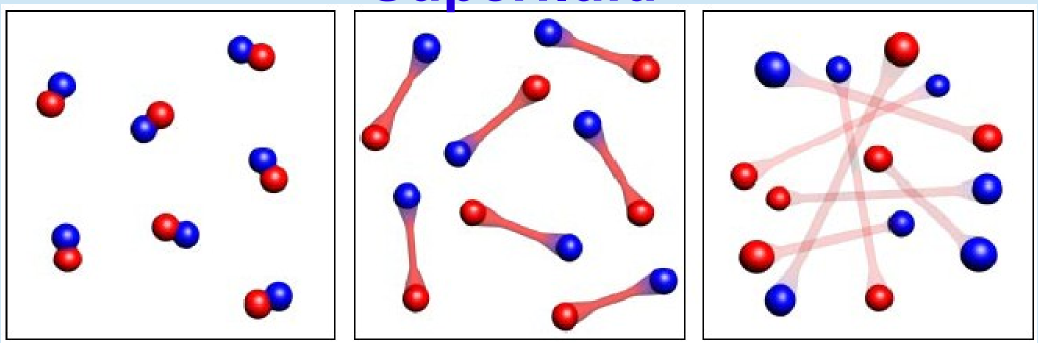
- Conclusions

Cold Fermi gases with tunable interaction

- Atomic Fermi gases: ^{40}K, ^{6}Li, ..

Crossover Superfluid

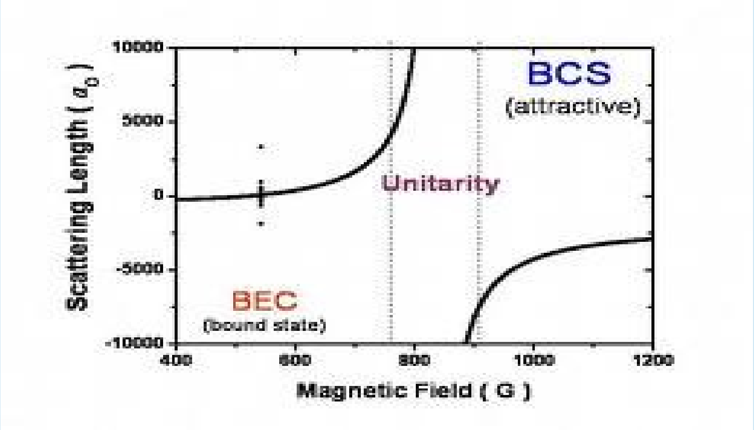
BEC of molecular pairs



Bosonic superfluid made of Cooper pairs

$a_F > 0$ $|a_F| = \infty$ $a_F < 0$

- The **BCS-BEC crossover** is attained by changing (with a Fano-Feshbach resonance) the s-wave scattering length a_F



(K.M. O'Hara et al., Science 2002)

Unitary limit: $|a_F| \to \infty$

Superfluid Hydrodynamics equations

At T=0 the collective dynamics of the Fermi gas is described by the extended (irrotational and inviscid) hydrodynamics equations

$$\frac{\partial}{\partial t}n + \nabla\cdot(n\mathbf{v}) = 0$$

$$m\frac{\partial}{\partial t}\mathbf{v} + \nabla\left[\frac{1}{2}mv^2 + U(\mathbf{r}) + \mu(n(\mathbf{r}), a_F) + T_{QP}\right] = 0$$

$$\mu(n; a_F) = \frac{d[n\varepsilon(n; a_F)]}{dn}$$

$$T_{QP} = -\lambda\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{n}}{\sqrt{n}}$$

Quantum Pressure

Equilibrium (v=0) profile consistent with DF:

$$E = \int d^3\mathbf{r}\; n(\mathbf{r})\left[\lambda\frac{\hbar^2}{8m}\frac{(\nabla n(\mathbf{r}))^2}{n(\mathbf{r})^2} + \varepsilon(n(\mathbf{r}); a_F) + U(\mathbf{r})\right]$$

Extended TF Functional

gradient term to describe inhomogeneities (surfaces, density waves,..)

Time Dependent Hydrodynamics equations

Superfluid order parameter $\psi(\mathbf{r},t) = \sqrt{n(\mathbf{r},t)}\; e^{i\theta(\mathbf{r},t)}$

Superfluid velocity $\mathbf{v}(\mathbf{r},t) = \frac{\hbar}{2m}\nabla\theta(\mathbf{r},t)$

Time dependent equation:

$$i\hbar\frac{\partial}{\partial t}\psi = \Big[-\frac{\hbar^2}{4m}\nabla^2 + 2U(\mathbf{r}) + 2\mu(|\psi|^2; a_F) + (1-4\lambda)\frac{\hbar^2}{4m}\frac{\nabla^2|\psi|}{|\psi|}\Big]\psi$$

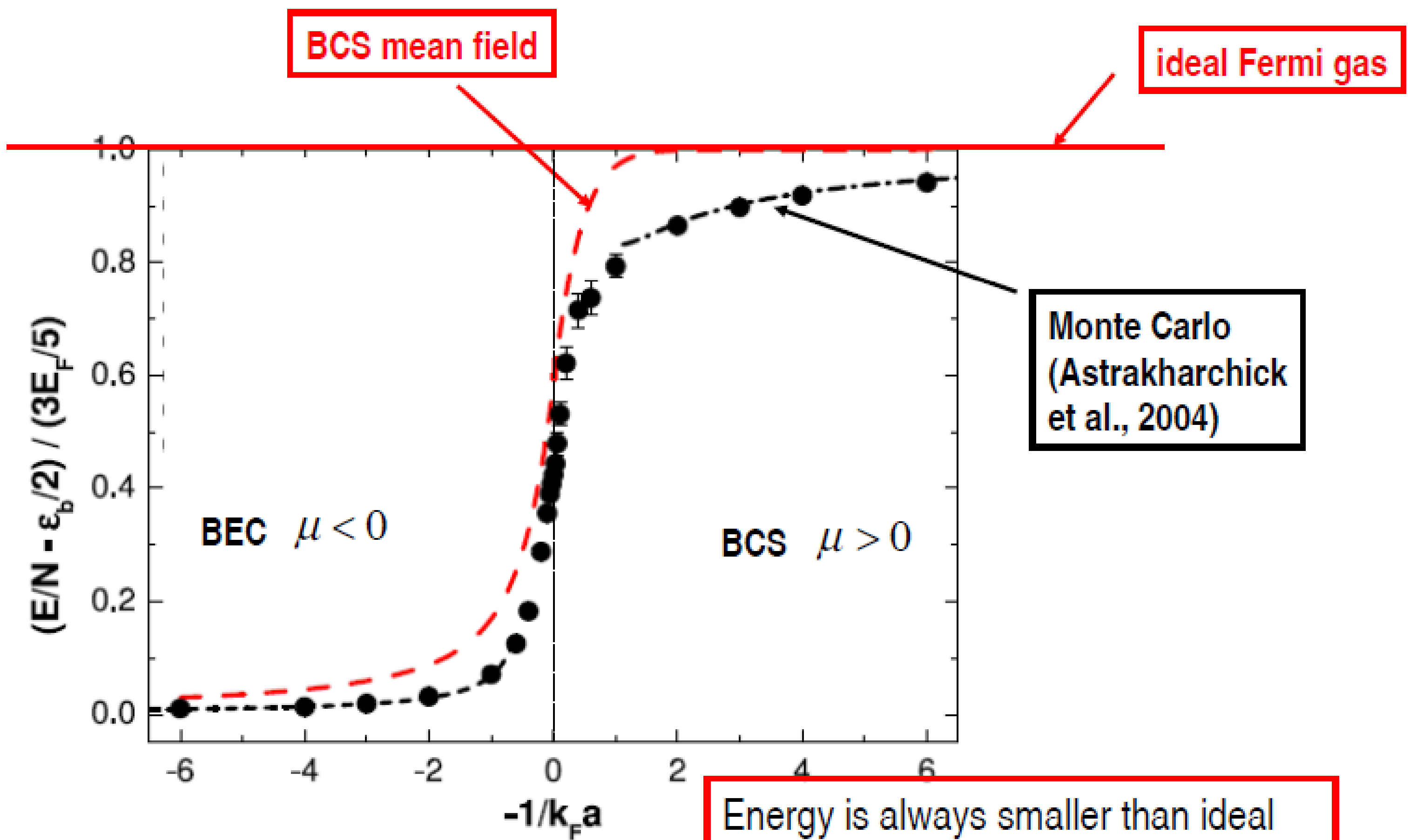
TD EFT

The bulk chemical potential (EOS) and the grad coefficient are input data

Fitting from MC data:
bulk EOS: N. Manini, LS, PRA (2005)
grad coefficient: LS, F. Toigo, PRA (2008); S.K. Adhikari, LS, NJP (2009)

Fermions

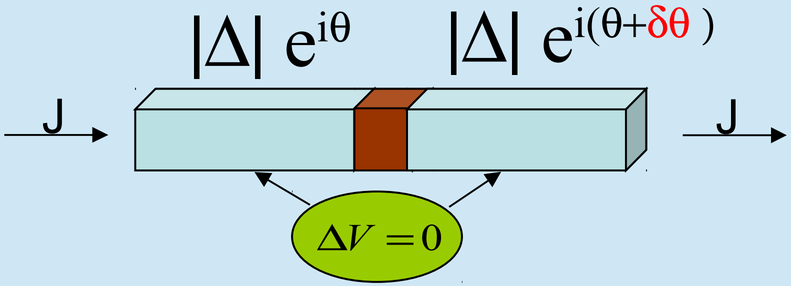
Equation of state along the BEC-BCS crossover



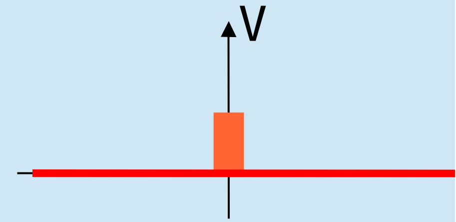
Energy is always smaller than ideal Fermi gas value. Attractive role of interaction along BCS-BEC crossover

Slide from Stringari (Crete 2007)

Stationary Josephson effect - 1



Two superconductors separated by a link: a current can flow with no potential drop



- The current through a **weak** link is related to the phase difference by Josephson's relation:

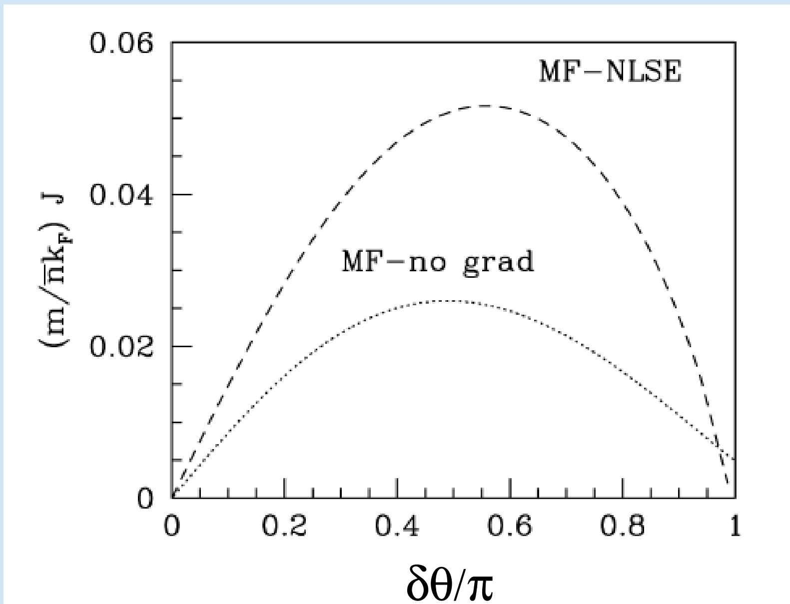
$$J = J_{max} \sin(\delta\theta)$$

- The same phenomenon occurs for two BECs separated by a potential barrier

TD Density functional calculations of Josephson effect in Fermi gas

- Gradient term necessary
- $\lambda = 1/4$ required to satisfy Josephson's relation

(F.Ancilotto, LS, F. Toigo, PRA 2009)



Stationary Josephson effect - 2

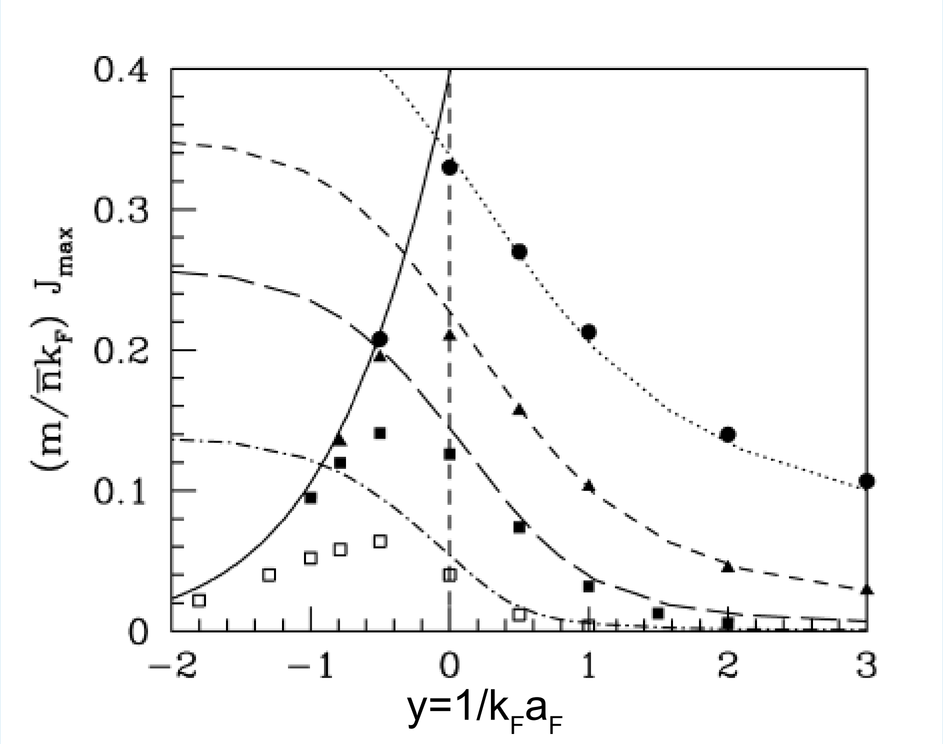
- For small barriers: $J_{max} = n\, v_{cr}$

TD density functional calculations

of Josephson effect in Fermi gas

reliable only

in the BEC side up to unitarity
(F.Ancilotto, LS, F. Toigo, PRA 2009)



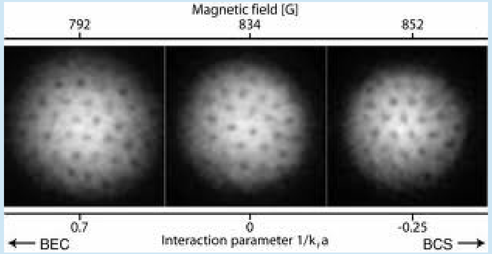
(comparison with Bogoliubov-de Gennes calculations by Spuntarelli et al., PRL 2007)

Unitary regime of a cold Fermi gas

$r_0 << n^{-1/3} << |a_F|$ both **dilute & strongly interacting!**

- **Universal behavior** (i.e. independent on the details of the interactions), with a characteristic length $d \propto n^{-1/3}$
- **Ground state energy** $E_0/N = \xi \frac{3}{5} \varepsilon_F \propto n^{2/3}$ (Bertsch, IJMP 2001)
- **Universal parameter** $\xi \simeq 0.4 - 0.45$ (Carlson et al., PRL 2003; Astrakharchik, Boronat, PRL 2004)
- **Superfluid** for **$T < T_c \sim T_F$** ("high T_c" superfluid!)

Proof: observation of quantized vortices on both sides of the Feshbach resonance



M.W.Zwierlein et al. Science (2006)

Time Dependent Hydrodynamics equations

Superfluid order parameter

$$\psi(\mathbf{r},t) = \sqrt{n(\mathbf{r},t)}\; e^{i\theta(\mathbf{r},t)}$$

Superfluid velocity

$$\mathbf{v}(\mathbf{r},t) = \frac{\hbar}{2m}\nabla\theta(\mathbf{r},t)$$

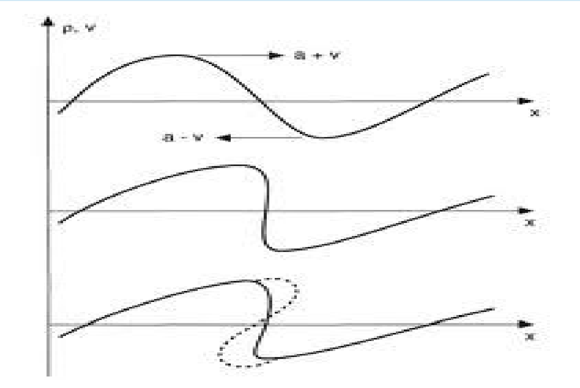
Time dependent equation:

$$i\hbar\frac{\partial}{\partial t}\psi = \Big[-\frac{\hbar^2}{4m}\nabla^2 + 2U(\mathbf{r}) + 2\xi\frac{\hbar^2}{2m}(3\pi^2)^{2/3}|\psi|^{4/3} + (1-4\lambda)\frac{\hbar^2}{4m}\frac{\nabla^2|\psi|}{|\psi|}\Big]\psi$$

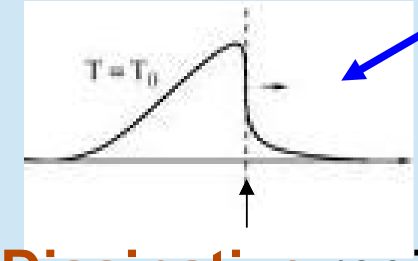
Describes cold Fermions HydroDynamics at unitarity

Shock waves in non-linear fluids: dispersion vs. dissipation

Non-linear wave dynamics

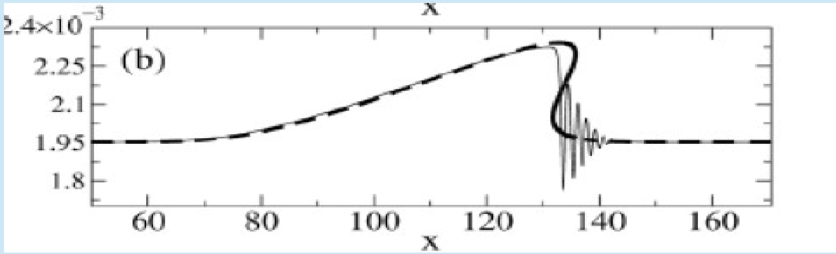


Avoiding the **"gradient catastrophe":**



Dissipative region

(**normal fluid**)



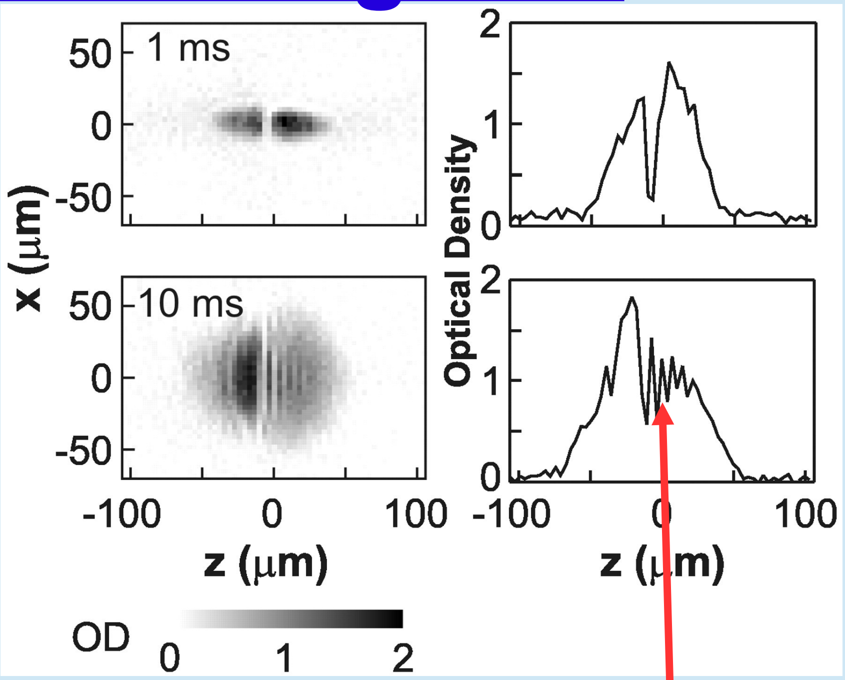
Dispersive waves (ripples, solitons...)

(**superfluid**)

Quantum shock waves in cold gases

- Shock waves in BEC observed in “hold, release & image” experiment

(Z.Dutton et al., Science 2001)

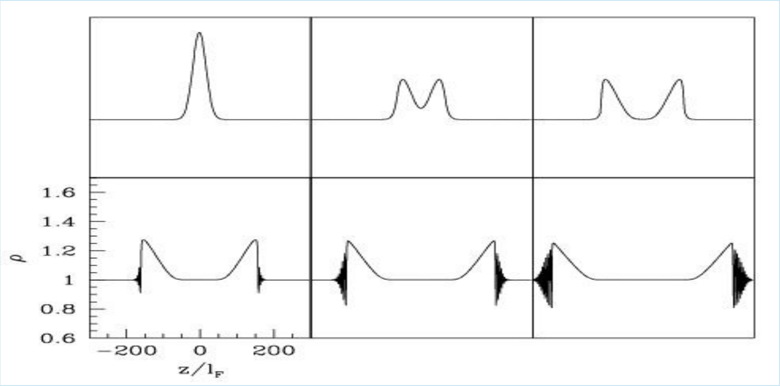


A train of ripples (“solitons”) is shedded

- Quantized vortices and solitons are nucleated when the spatial scale of induced density variations becomes of the order of the healing length

- Supersonic shock waves in Fermi superfluid

(LS, EPL 2011)



THE EXPERIMENT

A 50:50 mixture of the two lowest hyperfine states of ^{6}Li, is confined in a cigar-shaped laser trap and bisected by a blue-detuned laser beam which produces a repulsive potential.

The gas is cooled near a broad Feshbach resonance.

This produces two, spatially separated, atomic clouds containing a total of about 10^5 atoms per spin.

In the absence of the detuned beam the trapping potential is cylindrically symmetric, with a 16:1 ratio between the frequencies of the harmonic confinements in the radial and axial direction.

When the repulsive potential is abruptly turned off, the two clouds accelerate toward each other and collide in the trap.

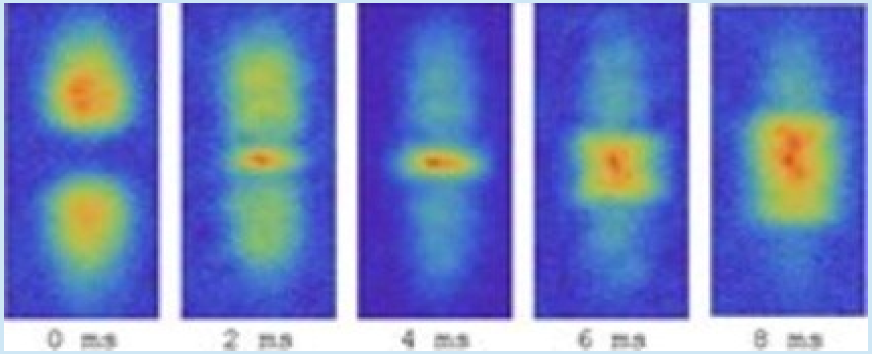
After a chosen time t the trap is removed,

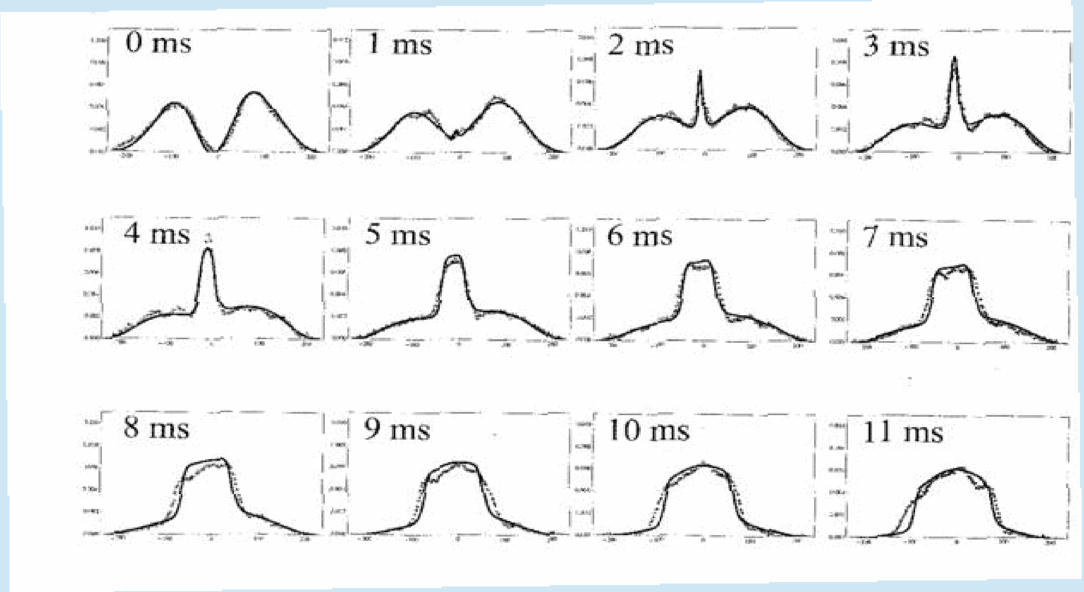
The atomic cloud expands for 1.5 ms and then it is imaged.

Shock waves in colliding Fermi clouds

(J.Joseph and J.Thomas, PRL 2011)

time →





N=10^5 ^{6}Li atoms at $T \sim 0.1T_F$

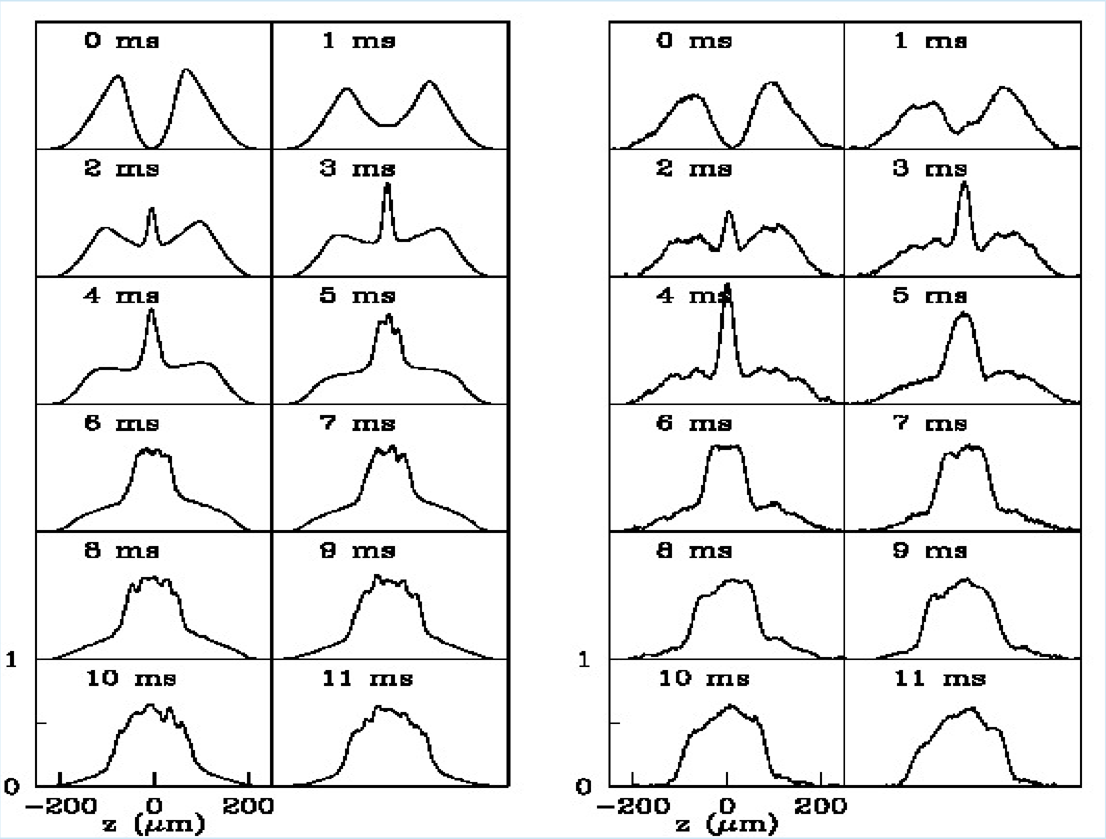
- Shock waves are observed
- A viscosity term is added to the hydrodynamics superfluid equations: the viscosity η used as an adjustable parameter

  $\eta \sim 10\hbar n$ fits the expt. data
- Conclusion: shock waves in Fermi gas are dissipative

But.....: The fraction of atoms in the non-superfluid component should be very small

Collision between Fermi clouds: theory vs. experiment

Theory*



Experiment

* Numerical solution of the time-dependent NLSE associated with the TDDFH equations

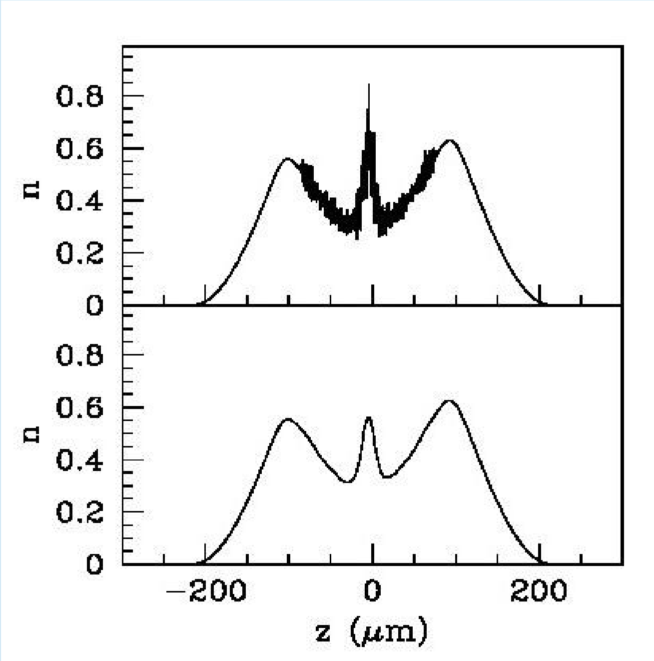
- Good agreement with experiment within purely dispersive dynamics (i.e. no dissipation)

- No adjustable parameters in the theory

(F.Ancilotto, LS, F. Toigo, PRA 2012)

Possible experimental observation of dispersive waves in Fermi gas

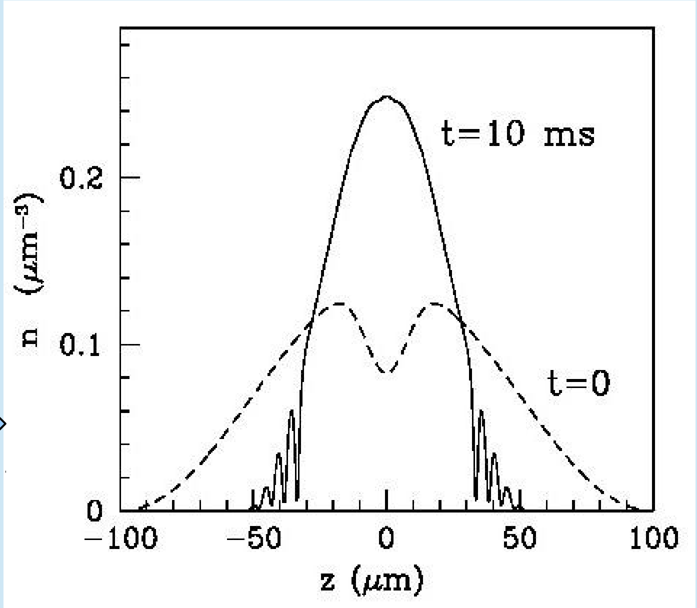
- Fast-collision: dispersive waves have too short wavelegth to be observed with typical experimental resolution (~5 μm)



Calculated density profile after 3 ms

After smoothing with experimental resolution

- dispersive shock waves should be observable instead by using a "soft-collision" expt. setup

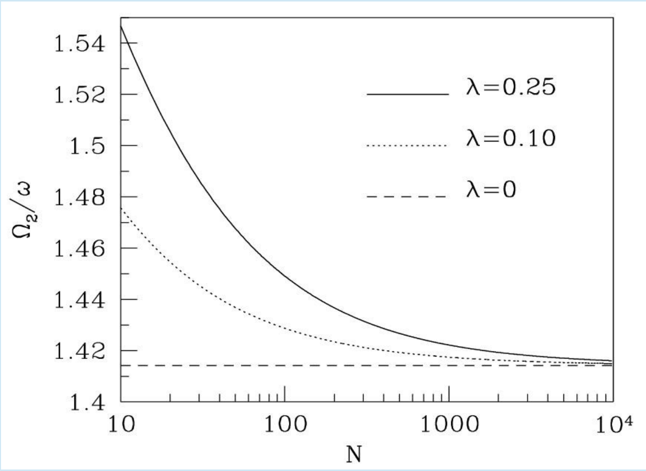


Role of the gradient term in large systems

(i) slowly varying density systems: the gradient term becomes less important as N increases

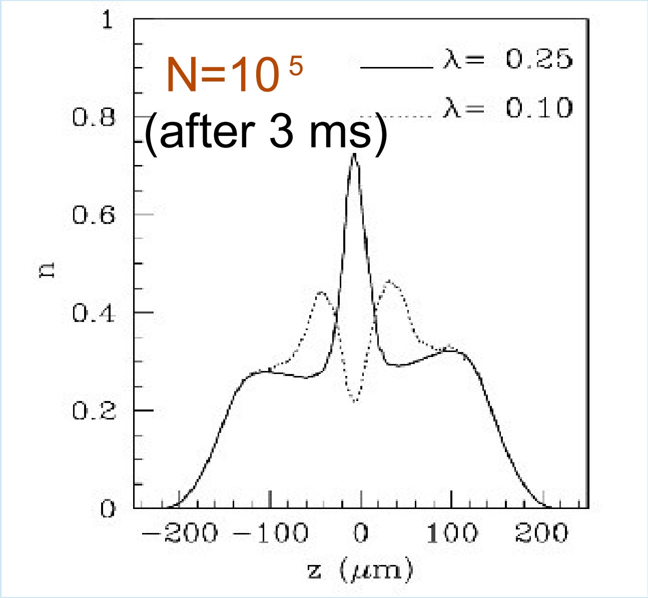
Example: small-amplitude quadrupole oscillations of a unitary Fermi gas under harmonic confinement

(F. Ancilotto, LS, F. Toigo., Laser Phys.Lett. 2010)



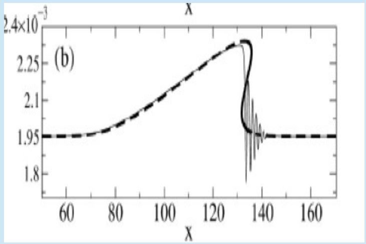
(ii) rapidly varying density (shock waves, ...): dispersion effects are important also for large systems

- By changing λ the long-time dynamics of the two colliding fermi clouds becomes completely different from the experimental one



What have we learned from this TDDF long-time dynamics of shock waves in the UFG at T=0?

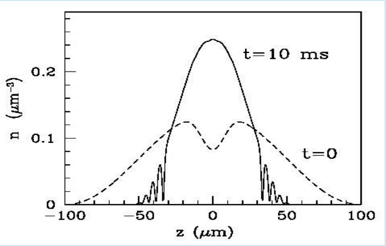
The regularization of the shock wave is purely dispersive



The quantum gradient term is important in the dynamics of large systems where large density gradients may arise

$$\lambda \frac{\hbar^2}{8m} \frac{(\nabla n)^2}{n^2}$$

Dispersive shock waves should be observable using a "soft" collision setup



Conclusions

- Time Dependent Density Functional method: simple yet accurate computational approach for large systems

- On the BEC side of the crossover, up to unitarity, a dynamical ETF model quantitatively reproduces low-energy dynamics of both microscopic BdG calculations and experiments.

- Low cost TDDFH calculations from an ETF functional faithfully simulate low energy TDSDFT simulations from an expensive microscopic BdG orbital-based functional.